



Learning and Teaching with Learning Trajectories: Counting

FACILITATOR GUIDE



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Isn't Counting Simple and Obvious? The Deep Mathematics of Counting

Before her fourth birthday, Abby was given five train engines. She walked in one day with three of them. Her father said, "Where's the other ones?" "I lost them," she admitted. "How many are missing?" he asked. "I have 1, 2, 3. So [pointing in the air] foouour, fiiiive . . . two are missing, four and five. [pause] No! I want these to be [pointing at the three engines] one, three, and five. So, two and four are missing. Still two missing, but they're numbers two and four."

Abby thought about counting and numbers—at least small numbers—abstractly. She could assign 1, 2, and 3 to the three engines, or 1, 3, and 5! Moreover, she could count the numbers. That is, she applied counting . . . to counting numbers! What are the ideas and skills that develop in such sophisticated counting? What do most young children know about counting? What more could they learn?

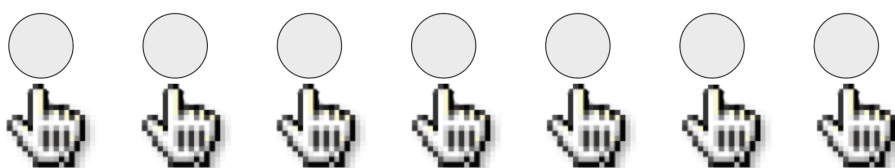
Early numerical knowledge includes many interrelated aspects, but four stand out at the very beginning. The first is small number recognition and subitizing, which is a separate but highly related learning trajectory. The other three pertain to counting. The second is learning the ordered list of number words to ten and beyond, or verbal counting. The third is enumerating objects, that is saying number words in correspondence with objects. The fourth is understanding that the last number word said when counting refers to how many items have been counted. There are many more, including solving problems with increasingly sophisticated counting strategies. Children learn these aspects, often separately through different kinds of experiences, but gradually connect them during the preschool years. {The following is from National Research Council, 2009 #3857}.

There are two different ways of thinking about the counting numbers: on one hand, they form an ordered list, and, on the other hand, they describe cardinality, that is, how many things are in a set. The notion of 1-to-1 correspondence bridges these two views of the counting numbers and is also central to the notion of cardinality itself. Let us see how.

Consider that numbers themselves are an abstraction of the notion of quantity because any given number quantifies an endless variety of situations. We use the number 3 to describe the quantity of three ducks, three toy dinosaurs, three people, three beats of a drum, and so on. We can think of the number 3 as an abstract, common aspect that all these limitless examples of sets of three things share. How can one grasp this common aspect that all sets of three things share? At the heart of this commonality is the notion of 1-to-1 correspondence. Any two collections of three things can be put into 1-to-1 correspondence with each other. This means that the members of the first collection can be paired with the members of the second collection in such a way that each member of the first collection is paired with exactly one member of the second collection, and each member of the second collection is paired with exactly one member of the first collection. For example, each duck in a set of three ducks can be paired with a single

egg from a set of three eggs so that no two ducks are paired with the same egg, no two eggs are paired with the same duck, and no ducks or eggs remain unpaired.

Counting is a way to make a 1-to-1 correspondence between each object (in which the objects can be any discrete thing, from a doll, to a drumbeat, to the idea of a unicorn) and a prototypical set, namely a set of number words. For example, when a child counts a set of seven bears, the child makes a 1-to-1 correspondence between the list 1, 2, 3, 4, 5, 6, 7 and the collection of bears. To count the bears, the child says the number word list 1, 2, 3, 4, 5, 6, 7 while pointing to one new bear for each number.



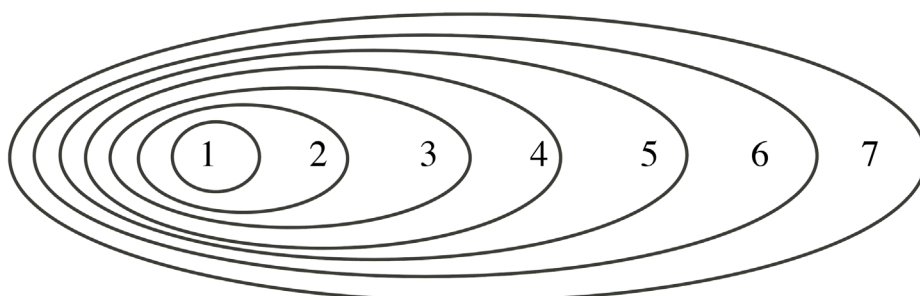
“one... two... three... four... five... six... seven. Seven in all!”

As a result, each bear is paired with one number, each number is paired with one bear, and there are no unpaired numbers or bears once counting is completed. The pairing could be carried out in many different ways (starting with any one of the bears and proceeding to any other bear next, and so on), but any single way of making such a 1-to-1 correspondence by counting establishes that there are seven bears in the set. A key characteristic of object counting is that the last number word has a special status, as it specifies the total number of items in a collection. For example, when a child counts a set of seven bears, the child counts 1, 2, 3, 4, 5, 6, 7, pointing to one bear for each number. The last number that is said, 7, is not just the last number in the list, but also indicates that there are seven bears in the set (i.e., cardinality of the set). Thus when counting the 7 bears, the counter shifts from a counting reference (to 7 as the last bear when counting) to a cardinal reference when referring to 7 as the number of bears in all. Counting therefore provides another way to grasp the abstract idea that all sets of a fixed number of things share a common characteristic—that when one counts two sets that have the same number of objects, the last counting word said will be the same for both

Another key observation about counting is that, for any given number in the list of counting numbers, the next number in the list tells how many objects are in a set that has one more object than do sets of the given number of objects. For example, if there are five stickers in a box and one more sticker is put into the box, then one knows even without counting them all again that there will now be six stickers in the box, because 6 is the next number in the counting list. Generally, each successive counting number describes a quantity that is one more than the quantity that the previous number describes. In a sense, then, counting is adding: Each counting number adds one more to the previous collection (see Figure 2-1). Of course, if one counts backward, then one is subtracting. These observations are essential for children’s early methods

of solving addition and subtraction problems. Also, each step in the counting process can be thought of as describing the total number of objects that have been counted so far.

Also, counting is the first and most basic and important algorithm. That is, most everything else in number, algebra, and beyond depend in some way on counting. Why is it an algorithm—a word usually used for ways to represent and process arithmetic with multidigit numbers (e.g., “column addition”)? Because an algorithm is a step-by-step procedure that is guaranteed to solve a specific category of problems. Counting is the first step-by-step procedure that children learn that solves certain problems—determining how many elements are in a finite set.



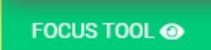
Children also learn to quickly tell how many there are in a collection if one is added or removed by counting up or down. Finally, children learn sophisticated counting strategies, such as counting on or counting backward to solve arithmetic problems, which we will describe in more detail in higher levels of this learning trajectory and especially in the adding and subtracting learning trajectory.

Starting Out: Where Are My Children on the LT for Counting?

Where Are My Children on the LT for Counting?	
Objectives <ul style="list-style-type: none"> • Explore the counting LT. • Be able to identify different levels of the counting LT. 	
Levels Addressed <ul style="list-style-type: none"> • Reciter • Corresponder • Reciter • Counter (Small Numbers) 	Big Ideas <ul style="list-style-type: none"> • Counting tells how many • Groups can be named with numbers • Math can be explored through materials
Facilitation Materials	
<ul style="list-style-type: none"> • Manipulatives of various type for counting • Exploring Counting Materials Activity Materials and Instruction 	

Directions

Sign on to LT² —<https://www.learningtrajectories.org/>

Click “Explore LTs” and then click the  button to highlight the age/grade level that represents most of the participants.

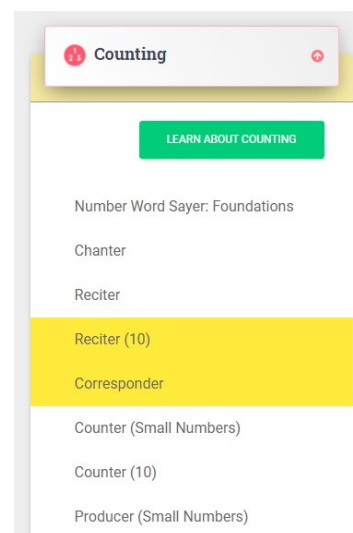
Show video: Show the *Introduction to Counting* video by clicking **Counting** and the Learn About button to get to the [Counting Trajectory](#) page.

Ask participants to go to view the developmental progressions on LT² and, after using the Focus Tool to highlight their age/grade group, explore several of the levels, *starting with the first highlighted level*, reading the descriptions and examples and watching videos. Remind them that this is just a gentle introduction—we will study them carefully throughout our time together.


- *What did you notice as you explored the levels of thinking from the developmental progression for counting?*
- *What differences do you notice between the developmental levels in terms of children’s thinking?*
- *Where do you think each of your children fall on the Counting LT?*

Ask participants to find a level and then an activity (click on the “Activities” button or just scroll down) that you’d like to try out in your classroom to build that level. Give participants time to explore the instructional activities and then share out.

Introduce the following activity, *Exploring Counting Materials*. This activity is a great starting point for teachers and parents. Through this activity teachers and parents can observe and assess each child’s current level of counting.



Exploring Counting Materials

 Exploring Counting Materials – Continuing learning to identify where students are on the counting LT.		
Materials	Trajectories	Objectives
Activity Materials and Instruction PDF — https://learningtrajectories.org/math-activities/exploring-counting-materials	Verbal Counting <i>Can the child recite numbers?</i> Object Counting <i>Can the child accurately count and produce small numbers?</i>	To explore mathematics in manipulatives and materials

Directions

Ask participants to read the activity directions for Exploring Counting Materials and use the introductory guiding question to facilitate discussion.

What is the main objective of the Exploring Counting Materials?

Do: Have participants practice the activity in groups of three. Allow time for all participants to rotate as the “teacher” with the other serving as a scribe and a “child”¹ who has chosen one LT level to exemplify.

What did you notice the “children” doing in this activity? What mathematical behaviors did you they demonstrate?

Reflect:

What do you hope to learn about your children?

What mathematical language might your children use? What mathematical language might you introduce or use in “mirror back” to children?

What types of questions would you be asking?

What objects or manipulatives would you choose? Could you continue this activity in different areas of the room (centers, tables, etc.)?

¹ Explain to teachers that the child should try to enact a level faithfully, but only “act out” the mathematical thinking. That is, this is a well-behaved child (!), not one who acts silly (falling off chair, try to eat manipulatives, etc.!)

Number Word Sayer: Foundations

When exploring this level, think about the purpose of the professional development session you are running and the aims that you want participants to walk away with at the end of the session. Below are different prompts to help you launch this activity. Choose and modify the prompts to best meet your session aims.

Ask participants to explore and discuss the information on this page.

Ask: What are children’s behaviors and ways of thinking that characterize this level?

Ask: What else might one see as children operate at that level?

Discuss where this level appears in GOLD.

Definition: Babies initially focus on one object or person at a time, yet they may at times hold two objects, one in each hand. Eventually, older infants demonstrate an understanding that there are different amounts of things. This then leads to some understanding that numbers represent “how many” and the understanding of words that identify how much.

Example: Apply their first number word, usually “two”. Use fingers to count a small number of items. Begin to use verbal number names.

Video: [Watch](#) a child performing at this level.

Though experiences can support development, note that foundational levels are not considered “targets” so we are not listing instructional activities with objectives to reach this level.

Objective 20: Uses number concepts and operations

a. Counts

Not Yet	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Verbally counts (not always in the correct order) • Says, “One, two, ten” as she pretends to count	Verbally counts to 10; counts up to five objects accurately, using one number name for each object • Counts to ten when playing “Hide and Seek” • Counts out four scissors and puts them at the table	Verbally counts to 20; counts 10-20 objects accurately; knows the last number states how many in all; tells what number (1-10) comes next in order by counting • Counts to twenty while walking across room • Counts ten plastic worms and says, “I have ten worms.” • When asked, “What comes after six?” says, “One, two, three, four, five, six, seven...seven.”	Uses number names while counting to 100; counts 30 objects accurately; tells what number comes before and after a specified number up to 20 • Counts twenty-eight steps to the cafeteria • When asked what comes after fifteen says “Sixteen.”	Counts to 120 to determine how many; uses skip counting by 2s, 5s, 10s, and 100s; begins counting forward at any number between 1 and 120; counts backwards from 20 • Says, “I can count to 50 really fast: 10, 20, 30, 40, 50” • Accurately counts from 115 to 120 beginning at 115 when asked to do so	Counts to 1,000 to determine how many; uses skip counting (2s, 5s, 10s, and 100s); begins counting forward at any number between 1 and 1000; switches between skip counts • Counts to 1,000 by 100s: “100, 200, 300...700, 800, 900, 1,000” • When asked to count to 200, begins counting by 2s but then changes to counting by 10s when she realizes it is taking too long	Counts to more than 1,000 using number word patterns (e.g., tens, teens) and skip counting; uses skip counting by 2s, 4s, 5s, 6s, 10s, and 100s • Begins counting “999, 1000, 1,110, 1,120, 1,130... 1,180, 1,190, 2,000” • Groups objects into sets of four then counts them: “Four, eight, twelve, sixteen, twenty, twenty-four, twenty-eight, thirty-two...”								

Note: GOLD, and to a large extent the Common Core State Standards, are based on, or borrowed from, our learning trajectories. They are basic benchmarks, so of course they do not include all the levels of our learning trajectories. That is what makes our learning trajectories more useful for teaching. Seeing the connections between GOLD and our learning trajectories, however, is valuable.



Chanter

When exploring this level, think about the purpose of the professional development session you are running and the aims that you want participants to walk away with at the end of the session. Below are different prompts to help you launch this activity. Choose and modify the prompts to best meet your session aims.

Ask participants to explore and discuss the information on this page.

Ask: What are children's behaviors and ways of thinking that characterize this level?

Ask: What else might one see as children operate at that level?

Discuss where this level appears in GOLD.

Subtrajectory: Verbal Counting

Definition: Chants number words in "sing-song" fashion and may run them together.

Example: Child knows number words, but not necessarily distinct words: "onetwothreefour".

Video: [Watch](#) a child performing at this level.


GOLD:

Objective 20: Uses number concepts and operations

a. Counts

Not Yet	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
		Verbally counts (not always in the correct order) <ul style="list-style-type: none"> Says, "One, two, ten" as she pretends to count. 	Verbally counts to 10; counts up to five objects accurately, using one number name for each object <ul style="list-style-type: none"> Counts to ten when playing "Hide and Seek" Counts out four scissors and puts them at the table 	Verbally counts to 20; counts 10-20 objects accurately; knows the last number states how many in all; tells what number (1-10) comes next in order by counting <ul style="list-style-type: none"> Counts to twenty while walking across room Counts ten plastic worms and says, "I have ten worms." When asked, "What comes after six?" says, "One, two, three, four, five, six, seven, seven." 	Uses number names while counting to 100; counts 30 objects accurately; tells what number comes before and after a specified number up to 20 <ul style="list-style-type: none"> Counts twenty-eight steps to the cafeteria When asked what comes after fifteen says "Sixteen." 	Counts to 120 to determine how many; uses skip counting by 2s, 5s, and 10s; begins counting forward at any number between 1 and 120; counts backwards from 20 <ul style="list-style-type: none"> Says, "I can count to 50 really fast: 10, 20, 30, 40, 50" Accurately counts from 115 to 120 beginning at 115 when asked to do so 	Counts to 1,000 to determine how many; uses skip counting (2s, 5s, 10s, and 100s); begins counting forward at any number between 1 and 1000; switches between skip counts <ul style="list-style-type: none"> Counts to 1,000 by 100s: "100, 200, 300...700, 800, 900, 1,000" When asked to count to 200, begins counting by 2s but then changes to counting by 10s when she realizes it is taking too long 	Counts to more than 1,000 using number word patterns (e.g., tens, tens) and skip counting; uses skip counting by 2s, 4s, 5s, 6s, 10s, and 100s <ul style="list-style-type: none"> Begins counting "999, 1000, 1,110, 1,120, 1,130 ... 1,180, 1,190, 2,000" Groups objects into sets of four then counts them: "Four, eight, twelve, sixteen, twenty, twenty-four, twenty-eight, thirty-two..." 							

Have participants explore activities at this level to understand what instruction looks like to help children achieve a Chanter level.

 Feel the Beat		
Materials	Trajectories	Objectives
Activity Materials and Instruction PDF — https://www.learningtrajectories.org/documents/1585588654693.pdf Video — https://www.learningtrajectories.org/math-activities/feel-the-beat	Counting	To achieve the Chanter level.

Ask participants to read the activity directions.

- *What is the main objective of Feel the Beat?*

Watch: Have participants watch the video.

- *What mathematical behaviors did the children demonstrate? What do you think the teacher may have learned about each child?*
- *What might you have done differently?*

Do: Have participants practice the activity in groups of three. Allow time for all participants to rotate as the “teacher.”

- *What did you notice the “children” doing in this activity? What mathematical behaviors did **they** demonstrate?*
- *What teaching ideas and strategies did you come up with?*

Reflect:

- *How and why do you think this activity would help children achieve the goal level of a Chanter?*
- *What mathematical language might be used? What types of questions might be appropriate?*
- *How will you implement this activity in your classroom? Whole groups? Small group?*
- *How many times do repeat a number in whole group? How would you differentiate?*



Reciter

When exploring this level, think about the purpose of the professional development session you are running and the aims that you want participants to walk away with at the end of the session. Below are different prompts to help you launch this activity. Choose and modify the prompts to best meet your session aims.

Ask participants to explore and discuss the information on this page.

Ask: What are children’s behaviors and ways of thinking that characterize this level?

Ask: What else might one see as children operate at that level?

Discuss where this level appears in **GOLD**.

Subtrajectory: Verbal Counting

Definition: Verbally counts with separate words up to at least five, but, not necessarily in the correct order above "five". May count objects or actions but not keep exact one-to-one correspondence with counting words.

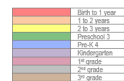
Example: "One, two, three, four, five, seven." If knows more number words than number of objects, rattles them off quickly at the end; if more objects "recycles" number words (inflexible list exhaustion).

Video: [Watch](#) a child performing at this level.

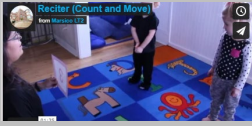
GOLD: Note this trajectory level captures some of the skills necessary between phases two and four of GOLD.

Objective 20: Uses number concepts and operations

Not Yet	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
		Verbally counts (not always in the correct order) • Says, "One, two, ten" as she pretends to count	Verbally counts to 10; counts up to five objects accurately, using one number name for each object • Counts to ten when playing "Hide and Seek" • Counts out four scissors and puts them at the table	Verbally counts to 20; counts 10-20 objects accurately; knows the last number states how many in all; tells what number (1-10) comes next in order by counting • Counts to twenty while walking across room • Counts ten plastic worms and says, "I have ten worms." • When asked, "What comes after six?" says, "One, two, three, four, five, six, seven...seven."	Verbally counts to 20; counts 10-20 objects accurately; knows the last number states how many in all; tells what number (1-10) comes next in order by counting • Counts to twenty while walking across room • Counts ten plastic worms and says, "I have ten worms." • When asked, "What comes after six?" says, "One, two, three, four, five, six, seven...seven."	Uses number names while counting to 100; counts 30 objects accurately; tells what number comes before and after a specified number up to 20 • Counts twenty-eight steps to the cafeteria • When asked what comes after fifteen says "Sixteen."	Counts to 120 to determine how many; uses skip counting by 2s, 5s, and 10s; begins counting forward at any number between 1 and 120; counts backwards from 20 • Says, "I can count to 50 really fast: 10, 20, 30, 40, 50" • Accurately counts from 115 to 120 beginning at 115 when asked to do so	Counts to 1,000 to determine how many; uses skip counting (2s, 5s, 10s, and 100s); begins counting forward at any number between 1 and 1000; switches between skip counts • Counts to 1,000 by 100s: "100, 200, 300...700, 800, 900, 1,000" • When asked to count to 200, begins counting by 2s but then changes to counting by 10s when she realizes it is taking too long	Counts to more than 1,000 using number word patterns (e.g., tens, teens) and skip counting; uses skip counting by 2s, 4s, 5s, 6s, 10s, and 100s • Begins counting "999, 1000, 1,100, 1,100, 1,150...1,180, 1,190, 2,000" • Groups objects into sets of four then counts them: "Four, eight, twelve, sixteen, twenty, twenty-four, twenty-eight, thirty-two..."						



Have participants explore the activity to understand what instruction looks like to help children achieve a Reciter level.



Count and Move Patterns

Materials	Trajectories	Objectives
Activity Materials and Instruction PDF — https://www.learningtrajectories.org/math-activities/count-and-move--patterns Video - https://www.learningtrajectories.org/math-activities/count-and-move--patterns	Counting	To achieve the Reciter level.

Ask participants to read the activity directions.

- *What is the main objective of Count and Move?*

Watch: Have participants watch the video.

- *What mathematical behaviors did the children demonstrate? What do you think the teacher may have learned about each child?*
- *How does this activity both practice extending verbal counting and lay the groundwork for correspondence?*
- *What movements would be most interesting to them? Most helpful?*
- *What might you have done differently?*

Do: Have participants practice the activity in groups of three. Allow time for all participants to rotate as the “teacher.”

- *What numbers might be best to “end” with—to emphasize those numbers? Why?*

Reflect:

- *How and why do you think this activity would help children achieve the goal level of a reciter?*
- *If several children have trouble counting past five consistently, what number(s) would you use in this activity? (Note For facilitators: If a child can count to 5 but mixes up numbers after five, then count to six until the child is comfortable with six always following five; in other words, one number past their successful counting.)*
- *When do you think the following formative assessment might be appropriate?*

Formative assessment:

If... Children have difficulty coordinating the motions of saying the number word and doing the motion,

Then... have children nod their heads instead of other motion (it is easier for them to control motions closer to their brains) and/or count slowly or to a smaller number.



Reciter (10)

When exploring this level, think about the purpose of the professional development session you are running and the aims that you want participants to walk away with at the end of the session. Below are different prompts to help you launch this activity. Choose and modify the prompts to best meet your session aims.

Ask participants to explore and discuss the information on this page.

Ask: What are children’s behaviors and ways of thinking that characterize this level?

Ask: What else might one see as children operate at that level?

Discuss where this level appears in **GOLD**.

Subtrajectory: Verbal Counting

Definition: Verbally counts to ten with some correspondence with objects, but may continue make errors (e.g. skipping, double counting).

Example: “One [points to first], two [points to second], three [starts to point], four [finishes pointing, but now is still pointing to the third object], five...nine, ten, eleven, twelve, 'firteen', fifteen...”

Video: [Watch](#) a child performing at this level.

GOLD:

Objective 20: Uses number concepts and operations

a. Counts

Not Yet	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
		Verbally counts (not always in the correct order) • Says, “One, two, ten” as she pretends to count		Verbally counts to 10; counts up to five objects accurately, using one number name for each object • Counts to ten when playing “Hide and Seek” • Counts out four scissors and puts them at the table		Verbally counts to 20; counts 10-20 objects accurately; knows the last number states how many in all; tells what number (1-10) comes next in order by counting • Counts to twenty while walking across room • Counts ten plastic worms and says, “I have ten worms.” • When asked, “What comes after six?” says, “One, two, three, four, five, six, seven, seven.”		Uses number names while counting to 100; counts 30 objects accurately; tells what number comes before and after a specified number up to 20 • Counts twenty-eight steps to the cafeteria • When asked what comes after fifteen says “Sixteen.”		Counts to 120 to determine how many; uses skip counting by 2s, 5s, and 10s; begins counting forward at any number between 1 and 120; counts backwards from 20 • Says, “I can count to 50 really fast: 10, 20, 30, 40, 50!” • Accurately counts from 115 to 120 beginning at 115 when asked to do so		Counts to 1,000 to determine how many; uses skip counting (2s, 5s, 10s, and 100s); begins counting forward at any number between 1 and 1000; switches between skip counts • Counts to 1,000 by 100s: “100, 200, 300, 700, 800, 900, 1,000!” • When asked to count to 200, begins counting by 2s but then changes to counting by 10s when she realizes it is taking too long		Counts to more than 1,000 using number word patterns (e.g., tens, teens) and skip counting; uses skip counting by 2s, 4s, 5s, 6s, 10s, and 100s • Begins counting “999, 1000, 1,110, 1,120, 1,130 ... 1,180, 1,190, 2,000!” • Groups objects into sets of four then counts them: “Four, eight, twelve, sixteen, twenty, twenty-four, twenty-eight, thirty-two...”	

Green	18 months to 1 year
Yellow	1 to 2 years
Orange	2 to 3 years
Red	3 to 4 years
Blue	Preschool
Purple	Kindergarten
Pink	1st grade
Brown	2nd grade

Have participants explore the activity to understand what instruction looks like to help children achieve a Reciter (10) level.



Baker's Truck

Materials	Trajectories	Objectives
Activity Materials and Instruction PDF — https://www.learningtrajectories.org/documents/1616384425982.pdf Video — https://www.learningtrajectories.org/math-activities/bakers-truck	Counting	To achieve the Reciter (10) level.

Ask participants to read the activity directions.

- *What is the main objective of the lesson?*

Teach participants the words and actions

One baker's truck drives down the street,

Filled with everything good to eat.

Two doors the baker opens wide. (Outstretch arms.)

Let's look at the shelves inside. (Cup hands around eyes to look.) What do you see? What do you see? Three big cookies for you and me! (Show three fingers.)"

Watch. Have participants watch the video.

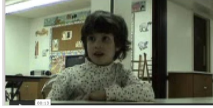
- *What mathematical behaviors did the children demonstrate? What do you think the teacher may have learned about each child?*
- *What did you notice the teacher doing in this activity? What might you have done differently?*
- *What mathematical behaviors did the children demonstrate? What do you think the teacher learned about each child?*

Do: Have participants practice the activity in groups of three. Allow time for all participants to rotate as the "teacher."

- *What did you notice the "children" doing in this activity? What mathematical behaviors did they demonstrate?*

Reflect:

- *How and why do you think this activity would help children achieve the goal level of Reciter (10)?*
- *Why this kind of finger play, as opposed to say, "Five Little Monkeys" at this point in children's development?*
- *How can you adapt the final number of cookies in the finger play to reinforce any number up to 10 (or beyond!) that you are teaching? How will you implement this activity in your classroom?*



Corresponder

When exploring this level, think about the purpose of the professional development session you are running and the aims that you want participants to walk away with at the end of the session. Below are different prompts to help you launch this activity. Choose and modify the prompts to best meet your session aims.

Ask participants to explore and discuss the information on this page.

Ask: What are children's behaviors and ways of thinking that characterize this level?

Ask: What else might one see as children operate at that level?

Discuss where this level appears in **GOLD**.

Subtrajectory: Verbal Counting

Definition: Keeps one-to-one correspondence between counting words and objects (one word for each object), at least for small groups of objects laid in a line. May answer a "how many?" question by recounting the objects, or violate 1-1 or word order to make the last number word be the desired or predicted word.

Example: "1, 2, 3, 4" (Child touches each object as the child counts the four objects)

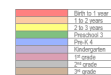
Video: Watch the [video](#) to see a child performing at this level.

GOLD:

Objective 20: Uses number concepts and operations

a. Counts

Not Yet	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Verbally counts (not always in the correct order) • Says, "One, two, ten" as she pretends to count			Verbally counts to 10; counts up to five objects accurately, using one number name for each object • Counts to ten when playing "hide and seek" • Counts out four scissors and puts them at the table		Verbally counts to 20; counts 10-20 objects accurately; knows the last number states how many in all; tells what number (1-10) comes next in order by counting • Counts to twenty while walking across room • Counts ten plastic worms and says, "I have ten worms." • When asked, "What comes after six?" says, "One, two, three, four, five, six, seven, seven."		Use number names while counting to 100; counts 30 objects accurately; tells what number comes before and after a specified number up to 20 • Counts twenty-eight steps to the cafeteria • When asked what comes after fifteen says "Sixteen."		Counts to 120 to determine how many; uses skip counting by 2s, 5s, and 10s; begins counting forward at any number between 1 and 100; counts backwards from 20 • Says, "I can count to 50 really fast: 10, 20, 30, 40, 50" • Accurately counts from 115 to 120 beginning at 115 when asked to do so		Counts to 1,000 to determine how many; uses skip counting by 2s, 5s, 10s, and 100s; begins counting forward at any number between 1 and 1,000; switches between skip counts • Counts to 1,000 by 100s: "100, 200, 300... 700, 800, 900, 1,000" • When asked to count to 200, begins counting by 2s but then changes to counting by 10s when she realizes it is taking too long		Counts to more than 1,000 using number word patterns (e.g., nine, twenty) and skip counting; uses skip counting by 2s, 4s, 5s, 6s, 10s, and 100s • Begins counting "999, 1000, 1,110, 1,120, 1,130... 1,180, 1,190, 2,000" • Groups objects into sets of four then counts them: "Four, eight, twelve, sixteen, twenty, twenty-four, twenty-eight, thirty-two..."	



Have participants explore the activity to understand what instruction looks like to help children achieve a Corresponder level.



Simon Says

Materials	Trajectories	Objectives
Activity Materials and Instruction PDF — https://www.learningtrajectories.org/documents/1573758699803.pdf Video — https://www.learningtrajectories.org/math-activities/simon-says	Counting	To achieve the Corresponder level.

Ask participants to read the activity directions.

- *What is the main objective of Simon Says?*
- *Do you see any possible other competencies this may build?*

Watch: Have participants watch the video.

- *What mathematical behaviors did the children demonstrate? What do you think the teacher may have learned about each child?*
- *What did you notice the teacher doing in this activity?*
- *What mathematical behaviors did the children demonstrate? Do you think they had some difficulties? What and why?*
- *What might you have done differently?*

Do: Have participants practice the activity in groups of their own choosing, taking turns giving commands.

- *Did you include some “silly” actions?*

Reflect:

- *How and why do you think this activity would help children achieve the goal level of Corresponder?*
- *What executive functions might be developed?*
- *How will you implement this activity in your classroom?*
- *What behaviors will help you decide what number to use next?*



Counter (Small Numbers)

When exploring this level, think about the purpose of the professional development session you are running and the aims that you want participants to walk away with at the end of the session. Below are different prompts to help you launch this activity. Choose and modify the prompts to best meet your session aims.

Ask participants to explore and discuss the information on this page.

Ask: What are children's behaviors and ways of thinking that characterize this level?

Ask: What else might one see as children operate at that level?

Discuss where this level appears in GOLD.

Subtrajectory: Verbal Counting

Definition: Accurately counts objects in a line to 5 and answers the "how many" question with the last number counted.

Example: "1, 2, 3, 4...four!"

Video: [Watch](#) a child performing at this level.

GOLD: This trajectory level captures some of the skills necessary between phases four and six of GOLD.

Objective 20: Uses number concepts and operations

Not Yet	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Verbally counts (not always in the correct order) • Says, "One, two, ten" as she pretends to count.	Verbally counts to 10; counts up to five objects accurately, using one number name for each object • Counts to ten when playing "Hide and Seek" • Counts out four scissors and puts them at the table	Verbally counts to 20; counts up to five objects accurately, using one number name for each object • Counts to ten when playing "Hide and Seek" • Counts out four scissors and puts them at the table	Verbally counts to 20; counts 10-20 objects accurately; knows the last number states how many in all; tells what number (1-10) comes next in order by counting • Counts to twenty while walking across room • Counts ten plastic worms and says, "I have ten worms." • When asked, "What comes after six?" says, "One, two, three, four, five, six, seven, seven."	Uses number names while counting to 100; counts 30 objects accurately; tells what number comes before and after a specified number up to 20 • Counts twenty-eight steps to the cafeteria • When asked what comes after fifteen says "Sixteen."	Counts to 120 to determine how many; uses skip counting by 2s, 5s, and 10s; begins counting forward at any number between 1 and 120; counts backwards from 20 • Says, "I can count to 50 really fast: 10, 20, 30, 40, 50" • Accurately counts from 115 to 120 beginning at 115 when asked to do so	Counts to 1,000 to determine how many; uses skip counting (2s, 5s, 10s, and 100s); begins counting forward at any number between 1 and 1000; switches between skip counts • Counts to 1,000 by 100s: "100, 200, 300, 700, 800, 900, 1,000" • When asked to count to 200, begins counting by 2s but then changes to counting by 10s when she realizes it is taking too long	Counts to more than 1,000 using number word patterns (e.g., tens, tens) and skip counting; uses skip counting by 2s, 4s, 5s, 6s, 10s, and 100s • Begins counting "999, 1000, 1,110, 1,120, 1,130... 1,180, 1,190, 2,000" • Groups objects into sets of four then counts them: "Four, eight, twelve, sixteen, twenty, twenty-four, twenty-eight, thirty-two..."							

Birth to 1 year
1 to 2 years
2 to 3 years
Preschool 3
Preschool 4
Kindergarten
1st grade
2nd grade
3rd grade

Have participants explore the activity to understand what instruction looks like to help children achieve a Counter (Small Numbers) level.



How Many in a Hand?

Materials	Trajectories	Objectives
Activity Materials and Instruction PDF — https://www.learningtrajectories.org/documents/1574108176765.pdf Video — https://www.learningtrajectories.org/math-activities/how-many-in-a-hand	Counting	To achieve the Counter (Small Numbers) level.

Ask participants to read the [activity directions](#).

- *What is the main objective of How Many in a Hand?*

Do: Have participants practice the activity in groups of three. Allow time for all participants to rotate as the “teacher.”

- *What if anything did you do to make this more fun? Did you think of any variations that do not change the mathematical thinking?*

Reflect:

- *How and why do you think this activity would help children achieve the goal level of Counter (Small Numbers)?*
- *What did you notice the “teacher” in your group doing in this activity? How did the teacher show the blocks to the children? What mathematical language was the “teacher” using? What mathematical language was the “teacher” reinforcing? What questions was the “teacher” asking?*
- *How did the “teacher” guide the children through this activity?*
- *How will you implement this activity in your classroom?*
- *How would you differentiate for a child that can answer “how many altogether” only up to 3? Or 7?*



Counter (10)

REMEMBER: For every new level, introduce it by going to that level on [LT]².

Ask participants to explore and discuss the information on this page.

Ask: What are children's behaviors and ways thinking that characterize this level?

Ask: What else might one see as children operate at that level?

Discuss where this level appears in **CCSSM** or the standard.

Subtrajectory: Verbal Counting

Definition: Counts arrangements of objects to 10. May be able to write numerals to represent 1–10. May be able to tell the number just after or just before another number, but only by counting up from 1. Verbal counting to 20 is developing.


Example: Accurately counts a line of 9 blocks and says there are nine.

What comes after 4? "1, 2, 3, 4, 5. 5!"

Video: [Watch](#) a child performing at this level.

CCSS: *Common Core State Standards Mathematics*

1.NBT.2.a 10 can be thought of as a bundle of ten ones — called a "ten."

 Memory Match (to 10)		
Materials	Trajectories	Objectives
Activity Materials and Instruction PDF — https://www.learningtrajectories.org/documents/1574096396454.pdf Video — https://www.learningtrajectories.org/math-activities/memory-match-to-10	Counting	To achieve the Counter (10) level.

Ask participants to read the activity directions.

What is the main objective of the lesson?

Watch. Have participants watch the video.

What mathematical behaviors did the children demonstrate? What do you think the teacher may have learned about each child?

What did you notice the teacher doing in this activity?

What question did the teacher ask after the children count to 10?

What is the importance of this question?

Reflect:

How and why do you think this activity would help children achieve the goal level of Counter(10)?

What if anything could you do to make this activity more fun? What directions did the “teacher” give the children? What mathematical language was the “teacher” using? What mathematical language was the “teacher” reinforcing? What types of questions was the “teacher” asking? How did the “teacher” guide the children through this activity?

How will you you adjust this activity for children with differing abilities?



Producer (Small Numbers)

REMEMBER: For every new level, introduce it by going to that level on LT²:

<https://www.learningtrajectories.org/math/counting/producer-small-numbers>

Ask participants to explore and discuss the information on this page.

Ask: What are children’s behaviors and ways thinking that characterize this level?

Ask: What else might one see as children operate at that level?

Discuss where this level appears in **WaKIDS** and **GOLD** (see below).

Definition: Counts out objects to 5. Recognizes that counting is relevant to situations in which a certain number must be placed.

Example: Produces a group of 5 objects.

Video: <https://www.learningtrajectories.org/math/counting/producer-small-numbers>

WaKIDS and GOLD: Note that “counting out” here is the *same* as what we call “Producer.”

Objective 20 Uses number concepts and operations

a. Counts

Not Yet	1	2	3	4	5	6	7	8	9
		Verbally counts (not always in the correct order) • Says, “One, two, ten” as she pretends to count		Verbally counts to 10; counts up to five objects accurately, using one number name for each object • Counts to ten when playing “Hide and Seek” • Counts out four scissors and puts them at the table		Verbally counts to 20; counts 10–20 objects accurately; knows the last number states how many in all; tells what number (1–10) comes next in order by counting • Counts to twenty while walking across room • Counts ten plastic worms and says, “I have ten worms.” • When asked, “What comes after six?” says, “One, two, three, four, five, six, seven...seven.”		Uses number names while counting to 100; counts 30 objects accurately; tells what number comes before and after a specified number up to 20 • Counts twenty-eight steps to the cafeteria • When asked what comes after fifteen, says “Sixteen.”	

Common Core State Standards

Not Yet	1	2	3	4	5	6	7	8	9
						K.CC.B.4. Understand the relationship between numbers and quantities; connect counting to cardinality. A. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. B. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.		K.CC.A.1. Count to 100 by ones and by tens. K.CC.A.2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1). K.CC.B.4. Understand the relationship between numbers and quantities; connect counting to cardinality. C. Understand that each successive number name refers to a quantity that is one larger.	



Cookie Game

Materials	Trajectories	Objectives
Activity Materials and Instruction PDF — https://www.learningtrajectories.org/documents/1599504468058.pdf Video — https://www.learningtrajectories.org/math-activities/cookie-game-producer-small-number	Counting	To achieve the Producer (Small Numbers) level.

Directions

Ask participants to read the activity directions.

What is the main objective of the lesson Cookie Game?

What are the multiple skills and concepts (ideas) this one activity might be teaching?

Watch. Have participants watch the video.

What mathematical behaviors did the children demonstrate? What do you think the teacher may have learned about each child?

What did you notice the teacher doing in this activity?

What questions does the teacher ask the children?

How does the teacher guide the children in the game?

In what ways do the children respond to the teacher and each other?

What might you have done differently?

Do: Have participants practice the activity in groups of three. Allow time for all participants to rotate as the “teacher.”

What if anything did you do to make this more fun or appropriate for your children?

How can you address the different math concepts in this activity?

Reflect:

How and why do you think this activity would help children achieve the goal level of Producer (small numbers)?

What did you notice the "teacher" from your practice doing in this activity? What directions did the "teacher" give the children? What mathematical language was the "teacher" using? What mathematical language was the "teacher" reinforcing? What questions was the "teacher" asking? How did the "teacher" guide the children through this activity?

How will you implement this activity in your classroom? Do you need to use cookies? Would this game work in a "choice" center?



Counter and Producer (10+)

REMEMBER: For every new level, introduce it by going to that level on LT²:

<https://www.learningtrajectories.org/math/counting/counter-and-producer-10>

Ask participants to explore and discuss the information on this page.

Ask: What are children’s behaviors and ways thinking that characterize this level?

Ask: What else might one see as children operate at that level?

Discuss where this level appears in **WaKIDS** and **GOLD** (see below).

Definition. Counts and counts out objects accurately beyond 10 (usually to 30 or more). Has explicit understanding of cardinality (numbers tell how many). Keeps track of objects that have and have not been counted, even in different arrangements. Writes or draws to represent 1 to 10 (then 20, then 30). Gives next number (usually to 20s or 30s) if allowed to generate a "running start." Recognizes errors in others' counting and can eliminate most errors in own counting (point-object) if asked to try hard.

Example. Counts a scattered group of 19 chips, keeping track by moving each one as they are counted.

Videos: <https://www.learningtrajectories.org/math/counting/counter-and-producer-10>

WaKIDS and GOLD: Children actually learn that the “last number word tells how many” back in the “Counter (Small Numbers)” level.

Objective 20 Uses number concepts and operations

a. Counts

Not Yet	1	2	3	4	5	6	7	8	9
		Verbally counts (not always in the correct order) • Says, “One, two, ten” as she pretends to count		Verbally counts to 10; counts up to five objects accurately, using one number name for each object • Counts to ten when playing “Hide and Seek” • Counts out four scissors and puts them at the table		Verbally counts to 20; counts 10–20 objects accurately; knows the last number states how many in all; tells what number (1–10) comes next in order by counting • Counts to twenty while walking across room • Counts ten plastic worms and says, “I have ten worms.” • When asked, “What comes after six?” says, “One, two, three, four, five, six, seven...seven.”		Uses number names while counting to 100; counts 30 objects accurately; tells what number comes before and after a specified number up to 20 • Counts twenty-eight steps to the cafeteria • When asked what comes after fifteen, says “Sixteen.”	

Common Core State Standards

Not Yet	1	2	3	4	5	6	7	8	9
						K.CC.B.4. Understand the relationship between numbers and quantities; connect counting to cardinality. A. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. B. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.		K.CC.A.1. Count to 100 by ones and by tens. K.CC.A.2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1). K.CC.B.4. Understand the relationship between numbers and quantities; connect counting to cardinality. C. Understand that each successive number name refers to a quantity that is one larger.	



Shopkeeper: Fill Orders

Materials	Trajectories	Objectives
Activity Materials and Instruction PDF — https://www.learningtrajectories.org/documents/1574111502049.pdf Video — https://www.learningtrajectories.org/math-activities/shopkeeper-fill-orders-counter-and-producer-10	Counting	To achieve the Counter and Producer (10+) level.

Directions

Ask participants to read the activity directions.

What is the main objective of Shopkeeper: Fill Orders?

Watch: Have participants watch the video.

What mathematical behaviors did the children demonstrate? What do you think the teacher may have learned about each child?

What did you notice the teacher doing in this activity? What might you have done differently?

What questions does the teacher ask and how does she ask the children to respond?

Do: Have participants practice the activity in groups of their choosing. Allow time for several participants to rotate as the “teacher.”

What if anything did you do to make this more fun?

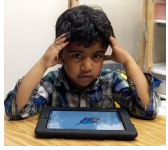
What variations can you add?

Reflect:

How and why do you think this activity would help children achieve the goal level of Counter and Producer (10+)?

What did you notice the “teacher” from your practice doing in this activity? What directions did the “teacher” give the children? What mathematical language was the “teacher” using? What mathematical language was the “teacher” reinforcing? What questions was the “teacher” asking? How did the “teacher” guide the children through this activity?

How will you implement this activity in your classroom?



Counter Backward from 10

REMEMBER: For every new level, introduce it by going to that level on LT²:

<https://www.learningtrajectories.org/math/counting/counter-backward-from-10>

Ask participants to explore and discuss the information on this page.

Ask: What are children's behaviors and ways thinking that characterize this level?

Ask: What else might one see as children operate at that level?

Subtrajectory. Verbal and Object

Definition. Counts backward from 10 to 1, verbally, or when removing objects from a group.

Example. "10, 9, 8, 7, 6, 5, 4, 3, 2, 1!"

Video. <https://www.learningtrajectories.org/math/counting/counter-backward-from-10>



Blooming Flowers

Materials	Trajectories	Objectives
Activity Materials and Instruction PDF — https://www.learningtrajectories.org/documents/1574357015390.pdf Video — https://www.learningtrajectories.org/math-activities/blooming-flowers	Counting	To achieve the Count Backward from 10 level.

Directions

Ask participants to read the activity directions.

What is the main objective of the lesson Blooming Flowers?

Watch: Have participants watch the video.

What mathematical behaviors did the children demonstrate? What do you think the teacher may have learned about each child?

What did you notice the teacher doing in this activity?

Can you think of other objects/situations to use in in this Blooming Flowers activity?

What might you have done differently?

Do: Have participants practice the activity in groups of their choosing. Allow time for several participants to rotate as the “teacher.”

What if anything did you do to make this more fun?

What were the variations of the Blooming Flowers activity used for backward counting?

Reflect:

How and why do you think this activity would help children achieve the goal level of Count Backward from 10?

What did you notice the “teacher” from your practice doing in this activity? What directions did the “teacher” give the children? What mathematical language was the

“teacher” using? What mathematical language was the “teacher” reinforcing? What questions was the “teacher” asking? How did the “teacher” guide the children through this activity?

How will you implement this activity in your classroom?



Counter from N (N + 1, N - 1)

REMEMBER: For every new level, introduce it by going to that level on LT²:

<https://www.learningtrajectories.org/math/counting/counter-from-n-n-1-n--1>

Ask participants to explore and discuss the information on this page.

Ask: What are children's behaviors and ways thinking that characterize this level?

Ask: What else might one see as children operate at that level?

Discuss where this level appears in **WaKIDS** and **GOLD** (see below).

Subtrajectory: Verbal and Object

Definition: Counts verbally and with objects from numbers other than 1 (but does not yet keep track of the number of counts). Immediately determines numbers just after or just before.

Example: Asked to "count from 6 to 8," counts "6, 7, 8!"

Asked, "What comes just before 7?" says, "Six!"

Videos: <https://www.learningtrajectories.org/math/counting/counter-from-n-n-1-n--1>

WaKIDS and GOLD: This and the next several levels are lumped together in the following description.

Objective 20 Uses number concepts and operations

a. Counts

Not Yet	1	2	3	4	5	6	7	8	9
		Verbally counts (not always in the correct order) • Says, "One, two, ten" as she pretends to count		Verbally counts to 10; counts up to five objects accurately, using one number name for each object • Counts to ten when playing "Hide and Seek" • Counts out four scissors and puts them at the table		Verbally counts to 20; counts 10–20 objects accurately; knows the last number states how many in all; tells what number (1–10) comes next in order by counting • Counts to twenty while walking across room • Counts ten plastic worms and says, "I have ten worms." • When asked, "What comes after six?" says, "One, two, three, four, five, six, seven...seven."		Uses number names while counting to 100; counts 30 objects accurately; tells what number comes before and after a specified number up to 20 • Counts twenty-eight steps to the cafeteria • When asked what comes after fifteen, says "Sixteen."	

Common Core State Standards

Not Yet	1	2	3	4	5	6	7	8	9
						K.CC.B.4. Understand the relationship between numbers and quantities; connect counting to cardinality. A. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. B. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.		K.CC.A.1. Count to 100 by ones and by tens. K.CC.A.2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1). K.CC.B.4. Understand the relationship between numbers and quantities; connect counting to cardinality. C. Understand that each successive number name refers to a quantity that is one larger.	



Counting Wand: What Comes Next?

Materials	Trajectories	Objectives
Activity Materials and Instruction PDF — https://www.learningtrajectories.org/documents/1574359489096.pdf Video — https://www.learningtrajectories.org/math/counting/counter-from-n-n-1-n--1	Counting	To achieve the Counter from N (N+1, N-1) level.

Directions

Ask participants to read the activity directions.

What is the main objective of Counting Wand: What Comes Next?

Watch. Have participants watch the video.

What mathematical behaviors did the children demonstrate? What do you think the teacher may have learned about each child?

What did you notice the teacher doing in this activity? What might you have done differently? How does the teacher engage the whole group?

How would you handle a child that is not yet at the Counter from N(N+1, N-1) during Counting Wand?

Do: Have participants practice the activity in groups of their choosing. Allow time for several participants to rotate as the “teacher.”

What if anything did you do to make this activity more fun?

What are the different ways the teachers engaged the participants?

Reflect:

How and why do you think this activity would help children achieve the goal level of Counter from N ($N+1$, $N-1$)?

What did you notice the "teacher" from your practice doing in this activity? What directions did the "teacher" give the children? What mathematical language was the "teacher" using? What mathematical language was the "teacher" reinforcing? What questions was the "teacher" asking? How did the "teacher" guide the children through this activity?

How will you implement this activity in your classroom? Could this activity be done in a smaller group when several children are ready?



Skip Counter by 10s to 100

REMEMBER: For every new level, introduce it by going to that level on LT²:

<https://www.learningtrajectories.org/math/counting/skip-counter-by-10s-to-100>

Ask participants to explore and discuss the information on this page.

Ask: What are children's behaviors and ways thinking that characterize this level?

Ask: What else might one see as children operate at that level?

Discuss where this level appears in **WaKIDS** and **CCSSM**.

Subtrajectory: Verbal and Object

Definition. Skip counts by tens up to 100 or beyond with understanding; e.g., "sees" groups of 10 within a quantity and count those groups by 10 (this relates to multiplication and algebraic thinking).

Example: "10, 20, 30...100."

Video: <https://www.learningtrajectories.org/math/counting/skip-counter-by-10s-to-100>

CCSSM: Common Core State Standards Mathematics:

K.CC.1 Count to 100 by ones and by tens.



Skip Counting Ourselves

Materials	Trajectories	Objectives
Activity Materials and Instruction PDF — https://www.learningtrajectories.org/documents/1574366766824.pdf Video — https://www.learningtrajectories.org/math-activities/skip-counting-ourselves	Counting	To achieve the Skip Counter by 10s to 100 level.

Ask participants to read the activity directions.

What is the main objective of the lesson Skip Counting Ourselves?

Watch: Have participants watch the video.

What mathematical behaviors did the children demonstrate? What do you think the teacher may have learned about each child?

What did you notice the teacher doing in this activity?

How is movement used in this activity?

What might you have done differently?

Do: Have participants practice the activity in groups of their choosing. Allow time for several participants to rotate as the “teacher.”

What if anything did you do to make this more fun?

Did the “teacher” find a variation?

How could you differentiate this activity?

Reflect:

How and why do you think this activity would help children achieve the goal level of Skip Counter by 10s to 100?

What directions did the “teacher” give the children? What mathematical language was the “teacher” using? What mathematical language was the “teacher” reinforcing? Was

the “teacher” asking questions? How did the “teacher” guide the children through this activity? How will you implement this activity in your classroom?



Counter to 100

REMEMBER: For every new level, introduce it by going to that level on LT²:
<https://www.learningtrajectories.org/math/counting/counter-to-100>

Ask participants to explore and discuss the information on this page.

Ask: What are children's behaviors and ways thinking that characterize this level?

Ask: What else might one see as children operate at that level?

Discuss where this level appears in **WaKIDS** and **CCSSM**.

Subtrajectory: Verbal

Definition. Counts to 100. Makes decade transitions (e.g., from 29 to 30) starting at any number.

Example: Start at 78 and keep counting. "...78, 79...80, 81..."

Video: <https://www.learningtrajectories.org/math/counting/counter-to-100>

CCSM: Common Core State Standards Mathematics

K.CC.1 Count to 100 by ones and by tens.



Number Flip

Materials	Trajectories	Objectives
Activity Materials and Instruction PDF — https://www.learningtrajectories.org/documents/1574362762868.pdf Video https://www.learningtrajectories.org/math-activities/number-flip-to-100	Counting	To achieve the Counter to 100 level.

Directions

Ask participants to read the activity directions.

What is the main objective of Number Flip?

What are the multiple skills and concepts (ideas) this one activity might be teaching?

Watch: Have participants watch the video.

What mathematical behaviors did the children demonstrate? What do you think the teacher may have learned about each child?

What did you notice the teacher doing in this activity?

How can a child who is not quite ready for Number Flip participate in this activity?

What might you have done differently?

Do: Have participants practice the activity in groups of three. Allow time for all participants to rotate as the “teacher.”

What if anything did you do to make this more fun?

What did the “teacher” do when a child made a mistake?

Reflect:

How and why do you think this activity would help children achieve the goal level of Counter to 100?

What did you notice the “teacher” from your practice doing in this activity? What directions did the “teacher” give the children? What mathematical language was the “teacher” using? What mathematical language was the “teacher” reinforcing? What questions was the “teacher” asking? How did the “teacher” guide the children through this activity? How will you implement this activity in your classroom?



Counter on Using Patterns

REMEMBER: For every new level, introduce it by going to that level on LT²:

<https://www.learningtrajectories.org/math/counting/counter-on-using-patterns>

Ask participants to explore and discuss the information on this page.

Ask: What are children's behaviors and ways thinking that characterize this level?

Ask: What else might one see as children operate at that level?

Discuss where this level appears in **WaKIDS** and **CCSSM**.

Subtrajectory: Strategy

Definition: Counts on, keeping track of a couple of counting acts, but only by using numerical pattern (spatial, auditory, or rhythmic) for adding 1 to about 3.

Example: "How much is 2 more than 5?" Child feels two "beats" as she counts, "5...6, 7!"

Video: <https://www.learningtrajectories.org/math/counting/counter-on-using-patterns>

CCSM. Common Core State Standards Mathematics

1.OA.5 Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).



How Many in the Box Now?

Materials	Trajectories	Objectives
Activity Materials and Instruction PDF — https://www.learningtrajectories.org/documents/1574361649784.pdf Video — https://www.learningtrajectories.org/math-activities/how-many-in-the-box-now-counter-on-using-patterns	Counting	To achieve Counter on Using Patterns level.

Directions

Ask participants to read the activity directions.

What is the main objective of How Many in the Box Now?

Watch. Have participants watch the video.

What mathematical behaviors did the children demonstrate? What do you think the teacher may have learned about each child?

What did you notice the teacher doing in this activity?

How are patterns used in counting?

What might you have done differently?

How does the teacher prompt and encourage the children?

Do: Have participants practice the activity in groups of three. Allow time for all participants to rotate as the “teacher.”

What if anything did you do to make this more fun? What variations can you add?

How can teachers perform this activity differently?

Reflect:

How and why do you think this activity would help children achieve the goal level of Counter on Using Patterns?

What did you notice the “teacher” from your practice doing in this activity? Was the “teacher” obvious when adding to the box? What mathematical language was the “teacher” using? What mathematical language was the “teacher” reinforcing? What questions was the “teacher” asking?

How will you implement this activity in your classroom?



Skip Counter

REMEMBER: For every new level, introduce it by going to that level on LT²:
<https://www.learningtrajectories.org/math/counting/skip-counter>

Ask participants to explore and discuss the information on this page.

Ask: What are children's behaviors and ways thinking that characterize this level?

Ask: What else might one see as children operate at that level?

Discuss where this level appears in **WaKIDS** and **CCSSM**.

Subtrajectory: Verbal and Object

Definition: Counts by fives and twos with understanding.

Example: "Child counts objects, "2, 4, 6, 8....20."

Watch the video

CCSM: Common Core State Standards Mathematics:
2.NBT.2. Count within 1000; skip-count by 5s, 10s, and 100s.

Skip Counter with Cubes		
Materials	Trajectories	Objectives
Activity Materials and Instruction PDF — https://www.learningtrajectories.org/documents/1574366315577.pdf Video — https://www.learningtrajectories.org/math-activities/skip-counting-with-cubes	Counting	To achieve the Skip Counter level.

Directions

Ask participants to read the activity directions.

What is the main objective of Skip Counter with Cubes?

Watch: Have participants watch the video.

What mathematical behaviors did the children demonstrate? What do you think the teacher may have learned about each child?

What did you notice the teacher doing in this activity?

How does the teacher interact with the children? How does the teacher prompt the children?

What might you have done differently?

Do: Have participants practice the activity in groups of three. Allow time for all participants to rotate as the “teacher.”

What if anything did you do to make this more fun? How did the “teachers” respond differently when you tried it?

Reflect:

How and why do you think this activity would help children achieve the goal level of Skip Counter?

What did you notice the “teacher” from your practice doing in this activity? What mathematical language was the “teacher” using? What mathematical language was the “teacher” reinforcing? What questions was the “teacher” asking?

How will you implement and differentiate this activity in your classroom?



Counter on Keeping Track

REMEMBER: For every new level, introduce it by going to that level on LT²:

<https://www.learningtrajectories.org/math/counting/counter-on-keeping-track>

Ask participants to explore and discuss the information on this page.

Ask: What are children's behaviors and ways thinking that characterize this level?

Ask: What else might one see as children operate at that level?

Discuss where this level appears in **WaKIDS** and **CCSSM**.

Subtrajectory: Strategy


Definition: Keeps track of counting acts numerically, first with objects, then by "counting counts."
Counts up to 4 or more from a given number.

Example: How many is 5 more than 6? "6...7 [puts up thumb], 8 [puts up a finger], 9 [puts up another finger] 10 [puts up another finger], 11 [puts up another finger], 11!"

Watch the video.

CCSM. Common Core State Standards Mathematics:

2.OA.2. Fluently add and subtract within 20 using mental strategies.

 Counter on with Objects		
Materials	Trajectories	Objectives
Activity Materials and Instruction PDF — https://www.learningtrajectories.org/documents/1574361079066.pdf Video - https://www.learningtrajectories.org/math-activities/count-on-with-objects	Counting	To achieve the Counter on Keeping Track level.

Directions

Ask participants to read the activity directions.

What is the main objective of the Counter on with Objects?

What other objects can you use in this activity?

Watch: Have participants watch the video.

What mathematical behaviors did the children demonstrate? What do you think the teacher may have learned about each child?

What did you notice the teacher doing in this activity? What might you have done differently?

What could you have done to assess the child that can add 10 and 4? Is he a Counter on with Correspondence or beyond? How can you be sure?

Do: Have participants practice the activity in groups of three. Allow time for all participants to rotate as the “teacher.”

What if anything did you do to make this more fun? Did you think of doing a variation?

Did one child pretend not to be able to count on with correspondence? How did the teacher react?

Reflect:

How and why do you think this activity would help children achieve the goal level of Counter on Keeping Track?

What mathematical language was the "teacher" using? What mathematical language was the "teacher" reinforcing? What questions was the "teacher" asking? How did the "teacher" guide the children through this activity? How will you implement and differentiate this activity in your classroom?



Counter of Quantitative Units/Place Value

REMEMBER: For every new level, introduce it by going to that level on LT²:

<https://www.learningtrajectories.org/math/counting/counter-of-quantitative-unitsplace-value>

Ask participants to explore and discuss the information on this page.

Ask: What are children's behaviors and ways thinking that characterize this level?

Ask: What else might one see as children operate at that level?

Discuss where this level appears in **WaKIDS** and **CCSSM**.

Definition. Understands the base-ten numeration system and place value concepts, including ideas of counting in units and multiples of at least tens and ones. When counting groups of 10, can decompose groups of 10, can decompose into 10 ones if that is useful. Understands value of a digit according to the place of the digit within a number. Counts unusual units, such as "wholes" when shown combinations of wholes and parts.

Example: Given sticks with bundles of 10 and some singles, counts by tens and then ones to determine the total number of sticks.

Watch the video.

CCSM. *Common Core State Standards Mathematics:*

2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.



Buying Candy Bars

Materials	Trajectories	Objectives
Activity Materials and Instruction — https://www.learningtrajectories.org/documents/1574360845599.pdf Video – https://www.learningtrajectories.org/math-activities/buying-candy-bars-counter-of-quant	Counting	To achieve the Counter of Quantitative Units/Place Value level.

Directions

Ask participants to read the activity directions.

What is the main objective of Buying Candy Bars?

What else can you as “money” to vary the activity?

Watch: Have participants watch the video.

What mathematical behaviors did the children demonstrate? What do you think the teacher may have learned about each child?

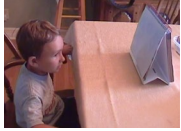
What did you notice the teacher doing in this activity? What might you have done differently?

Do: Have participants practice the activity in groups of three. Allow time for all participants to rotate as the “teacher.”

What if anything did you do to make this more fun? What kinds of “real world” variations can you add?

Reflect:

- *How and why do you think this activity would help children achieve the goal level of Counter of Quantitative Units/Place Value?*
- *What directions did the “teacher” give the children? What mathematical language was the “teacher” using? What mathematical language was the “teacher” demanding? What questions was the “teacher” asking?*
- *How will you implement and differentiate this activity in your classroom?*



Counter Beyond 100

REMEMBER: For every new level, introduce it by going to that level on LT²:
<https://www.learningtrajectories.org/math/counting/counter-beyond-100>

Ask participants to explore and discuss the information on this page.

Ask: What are children's behaviors and ways thinking that characterize this level?

Ask: What else might one see as children operate at that level?

Discuss where this level appears in **WaKIDS** and **CCSSM**.

Subtrajectory: Verbal and Object

Definition: Counts accurately beyond 100, recognizing the patterns of ones, tens, and hundreds.

Watch the Video.

CCSM: Common Core State Standards Mathematics:

2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.



Beavers and Sticks

Materials	Trajectories	Objectives
Activity Materials and Instruction PDF — https://www.learningtrajectories.org/documents/1574365700106.pdf Video - https://www.learningtrajectories.org/math-activities/beavers-and-sticks	Counting	To achieve Counter Beyond 100 level.

Ask participants to read the activity directions.

- *What is the main objective of the lesson Beaver and Sticks?*

Watch: Have participants watch the video.

- *What mathematical behaviors did the children demonstrate? What do you think the teacher may have learned about each child?*
- *What did you notice the teacher doing in this activity?*
- *How do the children respond differently in counting? How does the teacher respond?*
- *What might you have done differently?*

Do: Have participants practice the activity in groups of three. Allow time for all participants to rotate as the “teacher.”

- *What if anything did you do to make this more fun? What variations could you use?*

Reflect:

- *How and why do you think this activity would help children achieve the goal level of Counter Beyond 100*
- *What mathematical language was the “teacher” using? What mathematical language was the “teacher” reinforcing? What questions was the “teacher” asking? How did the “teacher” guide the children through this activity? Did the “teacher” respond to the child’s level of thinking?*



Number Conserver

REMEMBER: For every new level, introduce it by going to that level on LT²:
<https://www.learningtrajectories.org/math/counting/number-conserver>

Ask participants to explore and discuss the information on this page.


Ask: What are children's behaviors and ways thinking that characterize this level?

Ask: What else might one see as children operate at that level?

Definition: Consistently conserves numbers (i.e., believes number has been unchanged) even in the face of perceptual distractions such as spreading out objects of a collection.

Example: Counts 2 rows that are laid out across from each other and says they are the same. Teacher spreads out 1 row and says, "Both still have the same number; one is just longer."

Watch the video.

 Tricky Fox		
Materials	Trajectories	Objectives
Activity Materials and Instruction PDF — https://www.learningtrajectories.org/documents/1615919365127.pdf Video — https://www.learningtrajectories.org/math-activities/tricky-fox-number-conserver	Counting	To achieve the Number Conserver level.

Directions

Ask participants to read the activity directions.

What is the main objective of Tricky Fox?

Watch: Have participants watch the video.

What mathematical behaviors did the children demonstrate? What do you think the teacher may have learned about each child?

What did you notice the teacher doing in this activity?

What might you have done differently?

How can you relate the instructional video to the activity directions?

Do: Have participants practice the activity in groups of three. Allow time for all participants to rotate as the “teacher.”

What if anything did you do to make this more fun?

How can you change the story and implement this activity in different ways?

Reflect:

How and why do you think this activity would help children achieve the goal level of Number Conserver?

What did you notice the "teacher" from your practice doing in this activity? What questions was the "teacher" asking? Did the "teacher" try to trick the child? Could it work?



Counter Forward and Back (10s and 1s)

REMEMBER: For every new level, introduce it by going to that level on LT²:

<https://www.learningtrajectories.org/math/counting/counter-forward-and-back-10s-and-1s>

Ask participants to explore and discuss the information on this page.

Ask: What are children's behaviors and ways thinking that characterize this level?

Ask: What else might one see as children operate at that level?

Discuss where this level appears in **WaKIDS** and **CCSSM**.

Subtrajectory: Strategy

Definition: Counts "counting words" (single sequence or skips counts) in either direction starting at any number. Recognizes that decades sequences mirrors single-digit sequence. Switches between sequence and composition views of multidigit numbers easily.

Example: What's 4 less than 63?

"62 is 1 less, 61 is 2, 60 is 3, 59 is 4...so, 59."

Watch the video at this level.

CCSM: Common Core State Standards Mathematics:

2.NBT.5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.



Jumping on a Number Line

Materials	Trajectories	Objectives
Activity Materials and Instruction PDF — https://www.learningtrajectories.org/documents/1574359145030.pdf Video: https://www.learningtrajectories.org/math-activities/jumping-on-number-line	Counting	To achieve the Counter Forward and Back level.

Ask participants to read the activity directions.

What is the main objective of Jumping on a Number Line?

Watch: Have participants watch the video.

What mathematical behaviors did the children demonstrate? What do you think the teacher may have learned about each child?

What did you notice the teacher doing in this activity?

Would it have been appropriate to ask the children to skip count? Why or why not?

What might you have done differently?

Do: Have participants practice the activity in groups of three. Allow time for all participants to rotate as the “teacher.”

What if anything did you do to make this more fun? What kinds of counting varieties could you use?

Reflect:

How and why do you think this activity would help children achieve the goal level of Counter Forward and Back?

What did you notice the “teacher” from your practice doing in this activity? What questions was the “teacher” asking? How did the “teacher” guide the children through this activity? How will you implement this activity in your classroom?

Summary and Classroom Applications.

Directions

Ask participants to summarize what they have learned about counting.

Work together to select a level and have all participants try (possibly with just a couple of children) in their own classes/at home.

Work together to select a level and then an activity to have all participants try in their own classes/at home with their own children.