National Day Nurseries Association

Maths Champions

Developing Mathematical Thinking:

Shape, Space, Measure and Pattern

Course Summary

Module 1 - Shape and Space

Children's early experiences of shape, space and measures underpin mathematical understanding. Moving around requires babies to understand:

- The space they live in
- The objects within it.

They learn to:

- Predict what they will find
- Visualise spatially.

Research is showing that young children who are good at this can think mathematically and understand numbers using visual images.

Maths Pyramid for Babies and Young Children

According to this view of mathematical development, each type of activity builds on the one below, starting from babyhood. The broader the base of the pyramid, the sturdier the activity and understanding at the top of the pyramid will be. Of course, in reality, all the activities continue to develop alongside each other as children grow older.

(Adapted from Munn, 2009)

ACTIVITY: Discuss each area of Munn's Pyramid. How is this is supported in your setting?	
AREA	**DESCRIPTION**
Movement	Babies and children need plenty of opportunities and encouragement to engage in movement play. Some play and toys will encourage big movements and other small movements. Babies and children need time to develop both.
Problem Solving	Babies and children need time, space and encouragement to develop effective problem-solving strategies. Whilst playing young children may display repetitive behaviour. This should not be discouraged.
Spatial Understanding	Children need plenty of opportunities to: move up/down, left/right and front/back, forwards and backwards negotiate obstacles e.g. exploring outdoor environments, climbing frames, jigsaws create and define their own spaces - e.g. hiding, making a den.
Language	Children talk about what they are doing, Children need to hear language which describes and helps them to conceptualise what they are doing spatially. Children need lots of opportunities to talk and to be listened to.
Counting	Young children will begin by saying counting words without understanding but will develop understanding by hearing numbers used in meaningful and playful contexts. Counting games and songs can develop their understanding of counting words.
Number	Children should be exposed to a rich learning environment where there are plenty of opportunities to explore quantity. Adults should look out for emergent use of number knowledge as a problem solving tool.

ACTIVITY: What maths concepts is this baby learning about in this moment?

This baby is beginning to learn about…

- Space and shape, as he negotiates the 'straight' edges and rectangular appearance of the steps, and looks behind him to see the floor below from a new perspective
- Distance and speed measurements, as he climbs 'higher' and becomes more confident, so increases in pace, too
- Pattern, as he notices the actions needed to get to the next step and repeats this method to continue to climb, hearing different sounds and rhythms as he a hand hits the base and his foot thumps in a different way to push up again.

Children are learning spatial concepts as they move around in large and small spaces, observe the world about them and explore objects and materials. They realise that things look different from different viewpoints, remember where things are in terms of position and distance and understand how things move, developing mental images which allow them to predict what they will see or what might happen. If you play hiding games with very young children, they often hide by just putting their head out of view so they cannot see you but leave the rest of their bodies in plain view! Knowing what can be seen from where is an important aspect of spatial awareness, as is being aware of your own body.

Young children therefore need to move about and interact with objects in order to learn key aspects of spatial mathematics, including:

- Identifying positions of objects and directions of movement
- Selecting and combining shapes for a purpose, according to their properties.

Developing shape and space awareness

Being able to think spatially is important for everyday life, like visualising how to rearrange the furniture or following flat-pack diagrams. It is essential for skilled jobs like plumbing or architecture and careers in science, technology and design, which rely increasingly on 3D imaging. Recent research links children's early spatial abilities with their later attainment in mathematics. It seems that developing spatial visualising skills helps children to develop mental images such as a that of a number line and to understand visual representations of data, like graphs and diagrams.

Some children seem naturally good at spatial thinking, but research is showing that children can be taught these skills, rather than relying on 'natural ability'. Research suggests that there is no difference between boys and girls at pre-school in their ability to solve spatial problems, but boys seem to be better at shape rotation later in school. However, some children, including girls and those from less privileged backgrounds may lack early experiences to embed spatial reasoning skills in a variety of purposeful contexts and through play, so it is important that we provide opportunities to help all children to develop spatial reasoning.

ACTIVITY: Consider all of the opportunities you have in daily routines and play to support developing spatial reasoning skills. What other opportunities could you provide?

Spatial Awareness

Spatial awareness begins when babies move around and make sense of the space around them and the objects in it. They develop mental maps of where things are and how to get there.

Everyday activities, like tidying up and getting dressed, involve simple spatial awareness concepts, like ‘inside’ and ‘on top of’, and more complex ideas like stacking and ‘inside out’. Small world play, such as using roads and tracks, or farm layouts, hiding games and treasure hunts, obstacle courses and outings all help develop children’s spatial understanding. Talking about the route to the park will begin with the sequence of things and landmarks seen on the way. Later children will be able to visualise distances, positions and directions and begin to draw and understand pictorial maps.

TOP TIP: Early experiences such as construction activities and shape puzzles, like jigsaws, bead mazes and shape sorters, involve children in turning shapes round and fitting them together can help develop spatial problem-solving skills.

Talking during everyday activities helps children develop concepts of position and direction, using language like ‘on’, ‘under’, ‘inside’ and ‘up’ and ‘down’, ‘forwards’ and ‘back’. Some concepts seem more difficult for young children, like beside and between, and relative terms which depend on your viewpoint, like ‘in front of’ and ‘behind’.

Shape Awareness

Shape awareness involves combining shapes and understanding their properties, for instance knowing that flat shapes stack and curved shapes roll. Babies begin by exploring shapes and their characteristics by feel. Then they try and combine them by stacking them or fitting them inside each other. When children use shapes to make pictures and models, for instance using circles for eyes or wheels and rectangles for windows or beds, they show awareness of shape properties, such as curvedness or having corners. It helps children to identify and conceptualise properties if they learn words to describe them, such as 'straight', 'curved' or 'edge', including informal language such as 'pointy' or 'sloping'. Adults can help by discussing and describing what children have made.

TOP TIP: Discuss and describe what children create using both formal and informal shape language.

Although young children may become familiar with the names of shapes, such as ‘triangle’ and ‘rectangle’, only later will they really understand these and know there are many different kinds of triangles and that the category of rectangles includes squares. It is more important initially that they learn about shape properties through exploration, finding out, for instance, that curved shapes roll, flat surfaces stack with greater stability and right angles fit together.

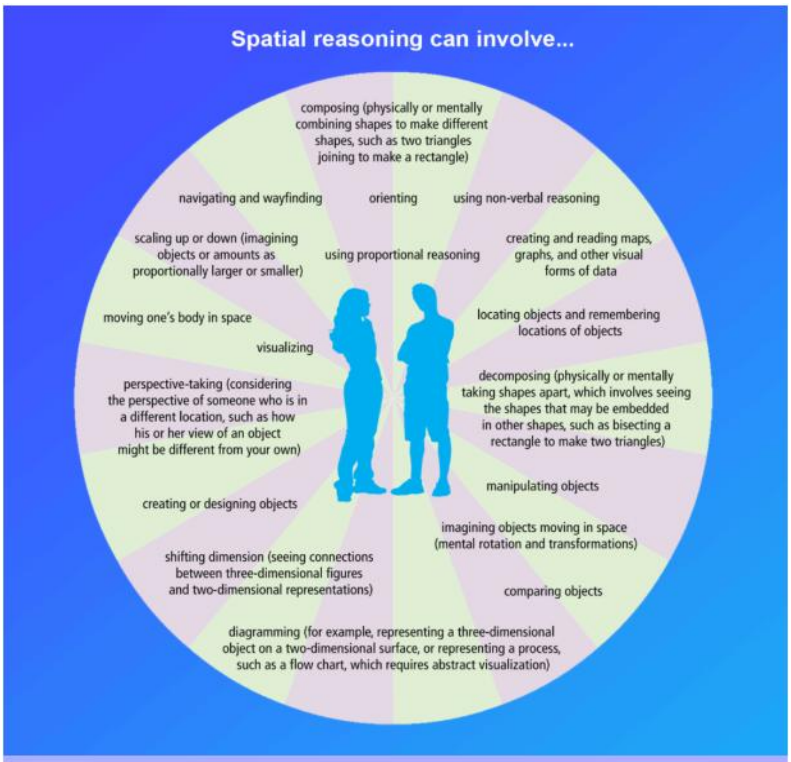
To support spatial awareness you could:

- Provide resources like shape sorters, insert and jigsaw puzzles, as well as picture and pattern-making provisions, will focus children's attention on properties of shapes as they rotate and fit them together
- Provide construction activities indoors and out, with a variety of materials of different sizes, including boxes, bricks and Lego, give children experience of fitting 3D shapes together, initially to make towers and later arches, then more complex enclosures, including large-scale den-making outdoors and obstacle courses
- Notice progress in the complexity of children's pictures and models, and provide occasional challenges, for instance to make a person with shapes, a bridge with arches, or a model from a picture.

Children's block play typically follows a progression in spatial ideas. Key developments are from just carrying blocks to stacking, then making towers or lines with identical blocks, to making enclosures and putting shapes together to make other shapes, with buildings of increasing complexity. Children's building strategies progress from trial and error, adding one block at a time, to planning: children with greater understanding of this process will make an arch by picking up the necessary blocks first, showing an advanced thought process. Later they will systematically make repeated units, such as towers of arches.

TOP TIP: In **Step 7** of the Maths Champions programme there is a **Core Activity** focussed on construction which provides more information about the progression of block play.

Spatial Reasoning



Paying attention to spatial reasoning, K-12 (Ontario Ministry of Education, 2014)

ACTIVITY: Discuss which aspects from this diagram are involved in young children's spatial play in your setting?

Module 2 - Pattern and Measures

Pattern

Pattern awareness has been found by researchers to be a better predictor than number understanding of children's overall maths achievement (Rittle-Johnson et al., 2016).

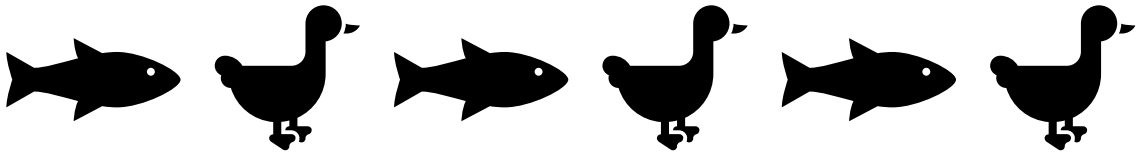
Teaching pattern awareness has been found to help children's mathematical thinking and has been particularly effective with children considered to be at a disadvantage. Patterns are all around us: on clothes, in nature, in music and speech, dance and repeated tasks, and also in times of the day and the seasons. Some young children are particularly 'pattern aware' and not only recognise, but spontaneously create patterns in their play, drawings and language.

The key idea in pattern awareness is noticing regularity or sameness. For instance, in the dice pattern of five dots, the dots are all the same size, they are equally spaced, four at each corner of a square, with one in the middle. The six-dot pattern has three dots on each side, equally sized and spaced.

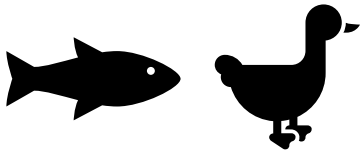
These regularities help to create a memorable image which can be recognised easily. Inviting children to copy patterns and talking about them helps children to notice the different features. Patterning in the early years involves recognising regular arrangements, often with symmetry, and repeating patterns.

This photo shows a young three-year-old creating a leaf and twig pattern with double sided sticky tape on a strip of card. She seems to have repeated consistently a pattern unit of a twig and leaves.

A pattern unit refers to the 'core pattern' that is repeated again and again. For example, in this pattern:



The pattern in its smallest, core form (before it starts to repeat) is:



The key to understanding patterns is recognising the unit of repeat or the smallest bit which you do 'over and over again' to create the repeated pattern.

Children can use a pattern unit to continue, copy and make their own patterns. Younger children usually find it hard to carry this on to the third repeat. They may describe a pattern as 'red, blue, red, blue, red, blue', while expert patterners talk about a 'red, blue' pattern, focusing on the smallest unit of repeat.

Usually, children start by making AB patterns, but may do this mechanically, for instance by using hands alternately. Young children can progress beyond AB patterns to ABC patterns (e.g. red, blue, yellow) while ABB (e.g. red, blue, blue) are harder still. Children can be challenged to continue a pattern, spot a deliberate mistake and fix it (which is a harder).

Leaf and twig pattern

Patterns are engaging because they can be done with all kinds of objects, like shoes, children or food and can involve art

activities like printing, with very pleasing results. Many action songs and rhymes have repeating patterns in movements and in words. Children enjoy spotting patterns and can make up their own with whatever they fancy. Patterns are accessible at a range of levels, and some children enjoy the challenge of making complex patterns for others to spot.

ACTIVITY: Discuss which opportunities you provide for children to experiment with and develop their understanding of pattern.

Repeating patterns progression of challenge:

- Continuing
- Copying
- Make your own
- Spot the mistake and fix it
- Identify the unit of repeat
- Making a pattern round a border.

Pattern units may also increase in challenge:

- AB, ABC, ABCD…
- AABB, ABB, ABBC …

TOP TIP: In **Step 7** of the Maths Champions programme there is a **Core Activity** focussed on pattern.

Measures

Measuring involves using numbers to compare qualities such as length or weight, using units, such as centimetres or grams. These are hard ideas for young children to understand, although they may have lots of home experience of measuring tools such as height charts, scales or speedometers.

Big and Small - Sian Smith

Children enjoy being involved in purposeful adult measuring activities, such as cooking or DIY, and even if they do not fully understand what is going on they will gain familiarity with tools and units. Comparing amounts underpins children's understanding of both numerical and spatial maths, so physical experiences and the language of comparing are important for their mathematical development.

Two year olds will compare things by general size, using

words like big and small.

Children progress to using terms for different attributes, like length or weight, using words like tall and short or heavy and light: practitioners can supply these more precise terms.

There are many words for length, depending on the context, e.g. height, distance and depth, with different comparative terms e.g. taller, further and deeper. With weight, children are often confused by the other meaning of light referring to colour, so it is important to provide dark objects which are light in weight and light-coloured objects which are heavy!

Children begin by comparing things directly. However, when comparing lengths, they may not realise they need to line up one end, so they may start a race in different places. Later they can compare indirectly, using a third object, like a stick or string, for instance to see if a table would fit in a space.

Weight is harder to compare than length, as it is an abstract concept like time, meaning we cannot see it visually and rely on measurements and feel. Although most adults nowadays use digital scales, early school activities usually involve balance scales to develop early ideas of comparison – being 'heavier' or 'lighter'. Children may not initially realise that the lower scale is heavier or that the aim is to balance the scales rather than to send one end crashing down! Again, discussion in meaningful contexts is important to help children understand this. A seesaw is a good preliminary experience for this.

Most comparison problems can be solved at different levels of difficulty:

- Directly, by lining up or pouring from one container to another
- Using an informal tool that does not have measure units on it but can be used for comparing measures e.g. a stick, a jug, balance scales
- Using repeated units, e.g. cubes, cupfuls (of the same size)
- Using a numbered measuring tool with units on it to measure with e.g. a tape measure or scales.

ACTIVITY: Discuss what activities and resources you provide to give children experience of different measures.

Capacity is harder to compare practically and verbally, with children often thinking that overflowing means a container holds more rather than less, so discussion and purposeful activities are important. To focus attention on this, it helps to have a purpose, such as, "Which bottle will hold enough for four teddies to have a cupful on their picnic?"

1 & 2 year olds

- Provide objects and recreate stories involving marked differences in size e.g. dolls and adult chairs, tiny and big bears, shoes, cups and bowls, blocks and containers and talk about big and small .
- Encourage comparison of size by ordering (e.g. stacking cups or Russian dolls).

3 & 4 year olds

- Provide containers of varying capacities (e.g. tall and thin, short and fat), items of markedly different weights and support children to compare; encourage pouring to compare containers and use phrases such as 'holds more 'holds less'
- Compare items of different lengths and provide opportunities to match lengths (e.g. make a dough worm as long as yours); talk about long and short
- Provide things with markedly different weights, including big light things and small heavy things e.g. sponges, balloons and oranges, pebbles; talk about heavy and light; provide seesaw and balance scale play
- In water play, sand, dough or mud kitchen, help children use direct comparison for size, length, weight, capacity and model comparative language e.g. longer/shorter; heavier/lighter; holds more/holds less.

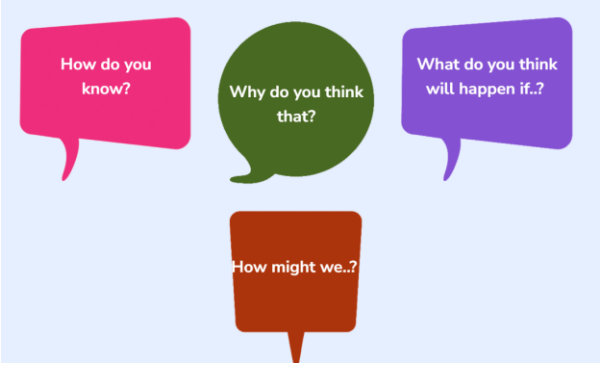
Module 3 - The Characteristics of Effective Learning and Sustained Shared Thinking

Sustained Shared Thinking

Open-ended problems provide opportunities for Sustained Shared Thinking (SST), identified by the Researching Effective Pedagogy in the Early Years (REPEY) study as the most effective pre-school teaching strategy (Siraj-Blatchford et al, 2002).

Sustained shared thinking involves having a genuine conversation, where both practitioner and child are solving a problem or developing an idea together and are focused on trying to understand each other. In doing so, the adult can get a better idea of the child's understanding, while providing opportunities for creative thinking. Interestingly, the REPEY study did not find any maths examples of sustained shared thinking. There may be several possible reasons for this - perhaps practitioners felt more skilled in developing conversations in other areas than maths. Perhaps mathematical questions tend to be closed, like "How many are there?" where the questions expect one specific, or limited, answers.

More open questions are more likely to elicit an extended response from older children to extend thinking and provide an opportunity to evoke curiosity from questions that inspire further thinking, such as:



However, these may also be too challenging for younger children to think through unsupported. Rather than asking questions, it is often more effective for adults to 'wonder' aloud, modelling being unsure and considering alternative approaches to a problem, inviting children to engage with your pondering through play together. This may look like…

Ludicrous suggestions or playful statements which are clearly wrong invite children to respond. Making deliberate mistakes is also an effective strategy, especially if a toy is making the mistakes, as this places children in the more powerful role of expert, not only engaging them in a playful situation but also to empower them to correct the adult, allowing adults and children to develop thinking alongside each other and build a meaningful relationship through playful maths.

As with other areas of learning, extended mathematical conversations depend on the child's interest and engagement in an activity. Sharing a purposeful activity, such as reading a book, solving a puzzle or trying to build something together is more likely to provoke relaxed but focused conversation. Some children may be more forthcoming with a friend doing the activity too.

ACTIVITY: Discuss what strategies you find effective in producing sustained mathematical conversations with children. Which others might you try?

TOP TIP: Download the **'Sustained Shared Thinking Techniques'** handout from **Module 3** of the course to support with this activity.

Characteristics of Effective Mathematical Learning and Teaching

Characteristics of Effective Learning describe behaviours children use in order to learn. To learn well, children must approach opportunities with curiosity, energy and enthusiasm. Effective learning must be meaningful to a child, so that they are able to use what they have learned and apply it in new situations. These abilities and attitudes of strong learners will support them to learn well and make good mathematical progress.

The Characteristics of Effective Learning have been identified as:

Playing and exploring

- Finding out and exploring
- Playing with what they know
- Being willing to 'have a go'.

Active learning

- Being involved and concentrating
- Keeping on trying
- Enjoying achieving what they set out to do.

Thinking creatively and critically

- Having their own ideas (creative thinking)
- Making links (building theories)
- Working with ideas (critical thinking)

(Reference: Birth to 5 Matters)

How do the Characteristics apply to mathematical learning?

Problem solving is a critical skill that can be applied to nearly every one of the characteristics listed, in a range of contexts. Problem solving is also a skill that is intrinsic to mathematical learning and lots of different maths-based play opportunities will prompt these characteristics, as will interesting and varied provision which prompts exploration.

Teaching strategies that can also support these characteristics include:

- Making 'silly' mistakes can provoke children into playing with what they know (for instance counting in the wrong order for children to correct the mistakes)
- Modelling finding a problem 'tricky' can encourage perseverance
- Being undecided about which way to approach a problem can prompt children to come up with new solutions (like using the hosepipe to fill a bath!)
- Asking about their thinking and opinions ("Why did you change your mind?" "Are you happy with that?)
- Valuing ideas ("Very good plan", "That's looking good")
- Encouraging then to test ideas and feel comfortable with making mistakes.

Conclusion

Developing knowledge in pattern, shape, spatial reasoning and measures are just as important to mathematical development as number sense and research suggests that they may actually be better predictors of later overall maths achievement than number itself.

This course has identified some of the strategies for effective teaching and providing rich learning environments with opportunities to explore and discover, as well as where we as professionals can encourage this thinking through methods like Sustained Shared Thinking.