

Article

Myths of Early Math

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Abstract: Myths about early education abound. Many beliefs people hold about early math have a grain of truth in them, but as a whole are not true—they are largely myths. But the myths persist, and many harm children. In this article, we address ubiquitous math myths that may be negatively affecting many young students. We conclude that avoiding the myths and listening to the findings of research and the wisdom of expert practice will serve both teachers and children well.

Keywords: at-risk students; classroom research; concrete manipulatives; early childhood; kindergarten; learning trajectories; literacy; mathematics; pedagogy; primary; play; preschool; social-emotional; teaching

“My daughter just does not understand math. I told her, ‘Don’t worry, honey. I was never good at math either.’”

“I know,” replied her friend. “Some people are math people, and some aren’t.”

Myths about math abound. You probably recognized two in the above conversation: first, that only a small number of “talented” people can succeed in mathematics, and second, subtle but equally as dangerous, the assumption that women do not usually succeed in mathematics. Many other myths exist, such as, “Math is one subject where there’s always one right answer and one way to get that answer.” Beliefs such as these have a grain of truth in them, but as a whole, are not true (see Appendix). Despite this lack of truth, myths persist, and many harm children. Let us look at some of the most important beliefs that negatively affect young children and separate the fact (if there is a grain of truth) from the myth for each.

1. “Early Math Is Just Counting.”

Fact. Counting is important. That is, *meaningful counting of quantities*, beyond simple verbal counting [1]. Counting is used to solve problems, for example, to obtain the correct number of paint brushes for the paint easel, or to ensure that everyone has the same number of snacks.

Myth. Many people make the assumption that early math should only focus on counting. However, even the youngest children can and should think mathematically in diverse and creative ways that go far beyond counting, e.g., [2,3]. They can develop the foundations of other topics, including arithmetic, geometry and spatial reasoning, measurement, and patterning.

For example, Bob Davis ([4], p. 154) tells this story of his 5-year-old daughter Alex, whose brother Paul was 3 years old. Alex tells Bob: “When Paul is 6, I’ll be 8; when Paul is 9, I’ll be 11; when Paul is 12, I’ll be 14 [she continues until Paul is 18 and she is 20].” Her father said, “How on earth did you figure all that out?” She replied, “It’s easy. You just go “three-FOUR-five” [saying the “four” very loudly, and clapping hands at the same time, so that the result was very strongly rhythmical, and had

a soft-LOUD-soft pattern], you go “six-SEVEN [clap]-eight”, you go “nine-TEN [clap!]-eleven” ([4] p. 154). This is a remarkable use of counting *and* patterning to solve an arithmetic problem.

Even before kindergarten, children have intuitions about arithmetic [1,5]. Consider an *infant* watching a tiny stage on which two dolls are placed. The curtain is drawn but a hand puts one more doll behind the screen. The screen opens revealing only 2 dolls! The 5-month-old looks *much longer* than when the opening reveals the expected number, 3 [6].

Moreover, young children can engage in mathematical processes, such as persevere in solving problems, reason and communicate about their reasoning, and search for and understand different kinds of patterns and structure [7–9].

2. “Children Need to Master Skills and Knowledge Before They Can Solve Problems.”

Fact. Children need to learn mathematical skills and concepts. They need to know about numbers to solve an addition problem such as “You have 4 blocks on the table. If you find two on the floor, how many will you have in all?”

Myth. It is *not* true that children cannot solve such problems until they know their addition facts [2,10]. Children can *put together what they know and invent ways to solve* such addition problems. A preschooler might put up four fingers on one hand, two on the other, then count them all. Listen to Jose: a kindergartner in Angela Andrew’s class: “It’s like foouuuuur, *five, six*. Six!” These children, just like Alex, are powerful problem solvers [11]. They have *modeled* the problem – another mathematic practice – and used what they do know to solve it.

3. “Young Children Must Sit Down and Learn Math. Sometimes You Just Have to Do Worksheets.”

Myth. Our memories of learning mathematics are often filled with images of workbooks and seat work. In contrast—for all ages, but especially for young children, good mathematics is about engagement and interest, not drudgery and drill [2,12,13]. High-quality early mathematics includes debating which child is bigger and drawing maps to a playhouse. It is about building with unit blocks and estimating and checking how many steps it is to the playground. It involves playing games, counting the dots on dice, and moving a game piece that many spaces [14–16].

4. “Time Spent on Math Is Time Taken Away from Play.”

Fact. If play is only defined as “free play”, there may be a few minutes less for such activity.

Myth. First, as the examples have shown, high-quality early math education takes a *playful* approach. Second, children, including toddlers [17], engage in spontaneous mathematics during almost half of every *minute of free play* [18], see also [19]. Teachers can build on such experiences. One teacher engaged in parallel play with children with playdough and raised questions regarding shapes and amount of dough. She told two boys she was “going to hide the ball” made of play dough, covered it with a flat piece, and pressed down. The boys said the ball was still there, but when she lifted the piece, the ball was “gone.” This delighted them and they copied her actions and discussed that the ball was “in” the “circle” ([20] pp. 31–32). Third, children who learn mathematics with intentional activities are more likely to engage in higher-quality *socio-dramatic play* during free-choice play time. Those in classrooms with an emphasis on mathematics were more likely to be engaged at a high-quality level than those in classrooms without this emphasis [21]. In this richer environment, individual children find more opportunities for meaningful engagement in free play. Thus, preventing children from experiencing intentional, structured mathematical experiences may deprive them of the joy and fascination of mathematics [2,22–24] as well as higher-quality play resulting from their increased mathematical knowledge. With good math, everyone wins.

5. “Time Spent on Math is Time Taken Away from Literacy and Social-Emotional Experiences.”

Myth. High-quality, good mathematics can involve moving, building, and playing [2,12,13,25]. So, children count the number of steps they climb and the number of times they hit a balloon to keep it afloat. Older preschoolers play card and board games, learning to share and play fair. Surprisingly, time spent on math *also contributes to literacy*. For example, working on shapes, puzzles, and other geometric skills builds “visual literacy” that contributes to better writing or composition, and even IQ scores years later [26]. In other studies, preschool mathematics programs promoted children’s language, literacy, and social-emotional development as much as literacy- or socially-focused programs did [27,28].

6. “Math Centers Are All You Need.”

Fact. Math learning centers, such as a table with a variety of manipulatives or a building blocks center, if well designed and supervised, probably contribute to children’s mathematics experiences.

Myth. Centers are insufficient by themselves. At no age level is it recommended that education should be entirely “child-centered” or “teacher-directed” [29]. Interaction with adults is key in all domains [30–32] and activities in small groups appear particularly effective [33,34]. However, mathematics, more so than other content areas, *builds*—concepts and skills are connected, abstracted, and curtailed, and become the object of thinking at a new level of mathematical thought e.g., [1,4,35–37]. Centers, as usually implemented, promote incidental learning at best and rarely build one mathematical idea on the next. Finally, only intentional activities focused on mathematics appear to make significant contributions to children’s learning [23,38].

7. “The Best Way to Teach Math is through “Teachable Moments.”

Fact. Teachable moments, handled well, can be wondrous and satisfying, e.g., [4,13].

Myth. However, teachable moments alone are far from adequate [12]. The teacher must carefully observe children and identify elements in the spontaneously-emerging situations that can be used to promote learning of mathematics [39]. However, there are serious problems with depending solely on this approach. For example, most teachers spend little time on careful observation necessary to find such moments [39,40], and spend little time with children during their free play [17]. Most teachers have a difficult time engaging children in tasks at their mathematical level [41]. Most teachers do not have applicable mathematics language and concepts at the ready, such as *relational* terms in mathematics [39,42]. Finally, even if professional development could address all of these issues, it is unrealistic for any teacher to see opportunities for multiple children to build multiple concepts consistently over the year [39].

8. “Young Children Always Need to Do Mathematics Concretely.”

Fact. Concrete objects, manipulatives, and situations are important for teaching and learning math. They can be essential in some contexts, and helpful in others [43–45]. In early phases of learning, without objects to count, children can make or understand quantities. For example, Les Steffe was working with Brenda. He covered four counters with a cloth, and then put two more next to it, then asked her, “How many in all?” Brenda tried to remove the cloth. When he held down the cloth and re-asked the question, she could not answer. She had to see the objects to count them [46].

Myth. Manipulatives do not “carry” mathematical ideas [47–49]. If kindergartners cannot use simple cubes to help them solve addition and subtraction problems, they likely have not learned a strategy to use the cubes to solve the problems. In this case, using a number line would be even more difficult [50]. Without concepts and strategies for how to use manipulatives, manipulatives alone are no help.

Stated differently, “... understanding does not travel through the fingertips and up the arm” ([47], p. 47). Children may require concrete materials to build meaning initially, but they need teachers

to help them reflect on their actions with manipulatives to do so [47–49]. They need teachers who can reflect on their students' representations for mathematical ideas and help them develop increasing sophisticated and mathematical representations.

Also, if manipulatives are treated as toys, they can actually interfere. Children were shown a location on a scale model of a room then asked to find same object in the actual room [51]. Children 2- and 3-years of age who played with a model of a room were less likely to be able to use it as a guide to find a hidden object in the room. Those who did not manipulate the model could use the small hidden object in it to find the actual object in the room. Children have to think of manipulatives as *mathematical*.

When children are able, move away from manipulatives [48,49]. This can seem counterintuitive, but the DeLoache study provided a rationale. In number work, although modeling necessitates manipulatives at some early levels of thinking, even preschoolers and kindergartners can use other representations, such as drawings and verbal or written symbols [10,18,52]. For example, in one study, kindergartners performed just as well without as they did with manipulatives in both accuracy and in their discovery of arithmetic strategies [53]. Consider Emily's explanation:

I find it easier not to do it [simple addition] with my fingers because sometimes I get into a big muddle with them [and] I find it much harder to add up because I am not concentrating on the sum. I am concentrating on getting my fingers right . . . which takes a while. It can take longer to work out the sum than it does to work out the sum in my head. [By "in my head", Emily meant that she imagined dot arrays. If that's what she liked, why didn't she just use those images? Why did she use fingers? She explains:] If we don't use our fingers, the teacher is going to think, "Why aren't they using their fingers . . . they are just sitting there thinking" . . . we are meant to be using our fingers because it is easier . . . which it is not.

([54], p. 35)

This opens a door some may wish to avoid exploring: Can educational technology also provide useful manipulatives and representations, even if they are not physically concrete? Yes—if high quality [29,55–57]. Computers can provide representations that are just as personally meaningful to students as physical objects and, paradoxically, research indicates that computer representations may be more manageable, "clean", flexible, and extensible than their physical counterparts [49]. Children who use physical and software manipulatives demonstrated a much greater sophistication in classification and logical thinking than did children that used physical manipulatives only [58]. Consider Kindergartner Chris, who is making shapes with a simplified version of Logo, a computer language for students. He has been typing "R" (for rectangle) and then two numbers for the side lengths. This time, he chooses 9 and 9. He sees a square and laughs.

Adult: Now, what do the two nines mean for the rectangle?

Chris: I don't know, now! Maybe I'll name this a square rectangle!

Chris uses his invented terminology repeatedly on succeeding days. He has constructed an intuitive notion of hierarchical classification, with squares being a subset of the class of rectangles defined as a rectangle that has all sides the same length. Computer manipulatives, *designed and implemented well* [29], have a multitude of psychological and pedagogical benefits [49].

9. Final Words

Teachers matter more than other factors, and teachers in the early years matter the most [59]. So, teachers of early mathematics have to use the best pedagogical strategies. Myths of math can limit teachers' visions of the breadth of such strategies. Every strategy, from play to direct instruction, can be educative or mis-educative. "Any experience is mis-educative that has the effect of arresting or

distorting the growth of further experience” ([60], p. 25). For example, mis-educative experiences resulting from inappropriate direct teaching may decrease sensitivity to creative applications of mathematical ideas or develop rote knowledge that narrows the range of further experiences. Conversely, child-centered education that rejects the structures or sequencing of subject matter content may be so disconnected as to limit later integrative experiences. “High-quality learning results from formal and informal experiences during the preschool years. ‘Informal’ does not mean unplanned or haphazard” ([61], p. 75). As John Dewey said, “Just because traditional education was a matter of routine in which the plans and programs were handed down from the past, it does not follow that progressive education is a matter of planless improvisation” ([60], p. 28).

Avoiding the myths and listening to the findings of research *and* the wisdom of expert practice, will serve teachers and children well. It will serve vulnerable children the most, as they benefit most from a balanced, research-based approach to early mathematics education, as a challenge and a joy [2,62].

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Appendix

Common misconceptions teachers hold about early mathematics education from [63].

1. Young children are not ready for mathematics education.
2. Mathematics is for some bright kids with mathematics genes.
3. Simple numbers and shapes are enough.
4. Language and literacy are more important than mathematics.
5. Teachers should provide an enriched physical environment, step back, and let the children play.
6. Mathematics should not be taught as a stand-alone subject matter.
7. Assessment in mathematics is irrelevant when it comes to young children.
8. Children learn mathematics only by interacting with concrete objects.
9. Computers are inappropriate for the teaching and learning of mathematics.

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