# Developmental pathways of early numerical skills during the preschool to school transition

PLEASE CITE THE PUBLISHED VERSION

https://doi.org/10.1016/j.learninstruc.2021.101484

PUBLISHER

Elsevier

VERSION

AM (Accepted Manuscript)

PUBLISHER STATEMENT

This paper was accepted for publication in the journal Learning and Instruction and the definitive published version is available at https://doi.org/10.1016/j.learninstruc.2021.101484.

# Developmental Pathways of Early Numerical Skills during the Preschool to School Transition

Abbie Cahoon, Camilla Gilmore & Victoria Simms

Accepted for publication in Learning and Instruction.

Author contact details are as follows:

*Corresponding author: Dr Abbie Cahoon; a.cahoon@ulster.ac.uk; Ulster University, UK; School of Psychology, Cromore Road, Coleraine, BT52 1SA.

Dr Camilla Gilmore; c.gilmore@lboro.ac.uk; Loughborough University, UK; Mathematics Education Centre, Schofield Building.

Dr Victoria Simms; v.simms@ulster.ac.uk; Ulster University, UK; School of Psychology, Cromore Road, Coleraine, BT52 1SA; Phone: 028 701 24395.

**Abstract**

Most longitudinal evidence explores the average level of development, suggesting that the relationships between a limited number of variables applies to all learners in the same way. This is the first longitudinal study that investigates multiple component numeric skills within a preschool population using a person-centered approach (i.e., a latent transition analysis), thus allowing for an investigation of different subgroup learning pathways of mathematical skills over time. 128 children aged 43 to 54 months (at Time 1) were tracked at three time points over 8 months encompassing the transition from preschool through to their first year of primary education. Findings suggest that there are five developmental pathways of mathematical learning with some groups of children making more rapid progress on entry to school than other groups. Those children in the low number skill pathway have a lower rate of growth than more advanced pathways, possibly due to a lack of understanding in cardinality. Findings highlighted the potential importance of language and working memory abilities on mathematical skills development over time.

**Keywords:** Latent transition analysis; developmental pathways of early numerical skills; longitudinal; preschool to school transition; multiple component numeric skills; domain-general and non-cognitive predictors

## Introduction

Children vary substantially in their mathematical skills prior to school-entry (Manolitsis, Georgiou & Tziraki, 2013). Understanding why some children start school more prepared to learn mathematics than their peers is critical, as early mathematics skills are among the strongest predictors of later academic achievement (Duncan et al., 2007). Most research suggests that proficiency in mathematics is not unitary, but actually comprises different component mathematics skills (e.g., Dowker, 2008; Lyons, Price, Vaessen, Blomert & Ansari, 2014). Commonly identified early numeracy skills include magnitude comparison (Batchelor, Keeble & Gilmore, 2015), numeral ordering (Lyons et al., 2014), cardinal principle knowledge (Condry & Spelke, 2008) and digit recognition (Wright, Martland & Stafford, 2006). However, we don't yet know how these skills are related over development.

There is some evidence that the development of these numeracy skills is influenced by a range of factors; this includes cognitive skills such as working memory (Peng, Namkung, Barne & Sun, 2016), language (Purpura, Hume, Sims & Lonigan, 2011) and sustained attention (Howse, Lange, Farran, & Boyles, 2003; McClelland, Morrison, & Holmes, 2000). There is also robust literature that indicates the wide-reaching impact of socioeconomic status (SES; Sammons et al., 2004) and the home numeracy environment (HNE; Dearing et al., 2012; Kleemans, Peeters, Segers & Verhoeven, 2012) on children's mathematical and development outcomes.

However, studies that investigate the development of numerical skills in early childhood (i.e. 3-5 years-old) commonly display at least one or more of the following limitations: (1) only a small number of component numerical skills are included, typically one or two (e.g. Purpura et al., 2011); (2) failure to consider the impact of domain-general skills or non-cognitive factors; (3) the use of cross-sectional methods (e.g. Batchelor et al., 2015); (4) use of analysis approaches that suggest all children follow the same developmental path.

Therefore, due to the lack of rich longitudinal data on numerical skill acquisition in this age group, it is difficult to ascertain the developmental pathway of early numeracy skills or identify factors associated with differences in growth from a person-centred approach. This information is important to allow teachers to provide appropriate, targeted support. Thus, the central purpose of this study is to examine developmental trajectories of a variety of basic numeracy skills that have been identified in previous studies to develop during this time. We first summarise recent findings on the development of early numerical skills and factors that influence this development, before considering research designs that are needed to move this literature forward.

**The development of early numerical skills**

During the preschool period children develop a range of basic numerical skills. Four skills have been identified as important indicators of a child’s early mathematical learning and development: *mapping* between number symbols and quantities, *order processing, cardinal principle* knowledge and *digit recognition*. These four skills capture important distinct elements of early arithmetic processing and development. The mapping and order processing tasks capture children’s proficiency with the semantic bases of number processing, i.e., magnitude and order information respectively. The *cardinal principle* knowledge task captures children’s counting knowledge: cardinal principle knowledge has been found to be more important than simple measures of rote counting. Finally, digit recognition is crucial for children to apply their understanding of number knowledge to abstract arithmetic. Overall, there is strong evidence for the development of these four basic numerical skills during preschool years. Below, we consider each of these skills in turn.

*Mapping* between number symbols and quantities is associated with mathematical skills in the preschool period and beyond (Mundy & Gilmore, 2009). Cross-notation comparison tasks have been used with preschool children to allow for the direct assessment of mapping between magnitude representations (e.g., dot arrays versus verbal number words). O*rder processing* has recently been considered as a crucial building block for the development of numerical skills (Lyons & Ansari, 2015). Order processing is knowledge of the associations between numbers (i.e., symbol-symbol relations rather than symbol-quantity relations). In a cross-sectional study involving Grades 1-6 (Lyons & Ansari, 2015) results showed that ordinality is a fundamental skill for children when processing numerical symbols and reinforcing more complex mathematical processes even at the earliest stages of education. Recent work has also established that order processing is also important in younger children, before formal schooling (AUTHORS, in prep).

A substantial body of the literature focuses on *cardinal principle* knowledge with preschoolers. In Give-N tasks (Wynn, 1990) children are asked to produce sets of items (e.g., Condry & Spelke, 2008; Wynn, 1990). A child demonstrates cardinal principle knowledge when counting a set of objects and the last word uttered correctly, in the correct order, expresses the number of items in the set. Thus, cardinal principle knowledge is necessary to give number words their meanings (Sarnecka & Wright, 2013). Cardinality has been shown to be important for preschool children's mathematics skills development as arguably the understanding of cardinality demonstrates the start of children's semantic representation of symbolic number (e.g., Batchelor et al., 2015; Bermejo, 1996; Lyons et al., 2014; Wagner & Johnson, 2011).

One of the most important skills is *digit recognition,* the child's ability to state the name of Arabic digits whether spoken or written (Wright, Martland & Stafford, 2006). *Digit recognition* has been found to be an important foundation for later mathematical development (Göbel, Watson, Lervåg & Hulme, 2014). However, being able to name Arabic digits does not necessarily imply that children show understanding of these symbolic codes (e.g., that the Arabic digit 4 means that there are four objects). These symbolic representations instead derive meaning from their connection to non-symbolic quantity information (such as dot arrays). Although, there is current debate about how and when this *mapping* association may arise (e.g., Li et al., 2018).

Overall, these four basic numeracy skills (i.e., digit recognition, symbol-quantity mapping, cardinal principle knowledge and order processing) have been identified as key indicators of mathematical learning from preschool onwards. However, most research is cross-sectional meaning there is a lack of understanding as to whether these four numeracy skills are inter-related in their trajectories of numeracy skill development. Cross-sectional evidence has shown relations among these skills for instance, Wagner and Johnson (2011) found a significant correlation between cardinal number knowledge and non-symbolic numerical discrimination with children aged 3-5 years old. Additionally, Slusser and Sarnecka (2011) using a cross-sectional approach found that children aged 2.5 to 4 years old, who understood cardinality, developed an understanding of the application of large number words to a variety of novel tasks (i.e. extend high-number words from one set to another based on their numerosity). Hence, in the current study, the longitudinal analyses that will be used on these four numeracy skills will describe the trajectories of skill development and thus reveal patterns of change in these numeracy skills over time.

**Individual differences in early numerical development**

Research suggests that there may not only be individual differences in children's level of understanding of mathematical skills but also individual differences in their development and the relation between them. Dowker (2008) revealed how some pre-schoolers were unable to procedurally count a set but understood how counting could be used to determine cardinality. Meanwhile, other children were capable of basic counting but lacked cardinal knowledge. Similarly, not all numerical skills follow the same pathway of mathematical development for every individual (Dowker, 2008) as some pre-schoolers may perform poorly on foundation tasks (e.g. counting) and yet may succeed on seemingly more complex tasks (e.g. cardinality). However, these studies are cross-sectional, hence this limits our understanding of the developmental trajectories of these numeracy skills. Some longitudinal studies exist however, for example Geary et al. (2018) who found that the age-of-acquisition at which children understood cardinal knowledge was central to their later number system knowledge, broader mathematical development and school readiness. Further, Chu, vanMarle & Geary (2016) found four quantitative competencies (i.e. cardinality, discrete quantity discrimination, numeral recognition, and nonverbal calculation) that predict later mathematics achievement, with intelligence and executive functions predicting growth in the four quantitative competencies, particularly across the first year of preschool. These studies however use traditional variable-centered analysis approaches which investigate the average level of development by assuming the relation between variables can be applied to all learners in the same way. This restricts the conclusions that can be drawn, and individual differences are consequently not explored (Hickendorff, Edelsbrunner, McMullen, Schneider & Trezise, 2017).

Nevertheless, one study that does investigate pre-schoolers (aged = 4- 7 years) developmental trajectories of number knowledge (i.e., Number Knowledge Test (NKT; Okamoto & Case, 1996)) from a person-centred approach is Garon-Carrier et al. (2018). Using semi-parametric mixture models, they found a 4-pathway model, Low-Increasing (10% of the children), Moderate-Increasing (39%), Moderate-Fast Increasing (32%) and High- Increasing (19%) groups, with the Low-Increasing group persistently scoring lower than the other groups throughout the years. However, Garon-Carrier et al. (2018) did not investigate component numerical skill development, whereas the current study will examine the patterns of numeracy skill development by looking at four component numerical skills.

**Predictors of early numerical development**

Many different predictors of early numerical development have been identified including *working memory*, *vocabulary* and *sustained attention* which have consistently been linked to the development of children's numerical skills (e.g. Bull, Espy & Wiebe, 2008; Jordan, Hanich, & Kaplan, 2003; McClelland et al., 2000; Purpura et al., 2011; Xenidou-Dervou et al., 2018). Working memory plays a vital role in children's early mathematical learning (e.g. Bull, Espy & Wiebe, 2008; Xenidou-Dervou et al., 2018) as working memory is involved when manipulating information such as when connecting quantities to number words (Vukovic & Lesaux, 2013). Xenidou-Dervou et al. (2018) using a latent growth model, found that preschool children's working memory capacities, IQ score and counting skills (i.e. the four counting subscales from the Early Numeracy Test-Revised; number words, structured counting, resultative counting and general understanding of numbers and using the counting system in everyday life) were all unique predictors of the children's early mathematics achievement.

Vocabulary knowledge is involved when counting and solving numeracy problems (Jordan et al., 2003). Purpura et al. (2011) identified that early print knowledge and *vocabulary* accounted for a unique variance in the prediction of early numeracy development. Previous research has reported that individuals with reading or language problems perform poorly on arithmetic tasks (e.g. multiplication and fact retrieval) compared to those without these problems (Moll, Snowling, Göbel & Hulme, 2015). Some research even claims that mathematics is a language-dependent skill (Abedi & Lord, 2001, De Smedt, Taylor, Archibald & Ansari, 2010, Kim, Ferrini-Mundy & Sfard, 2012).

*Sustained attention* skills have also been observed to be associated with later academic achievement, independent of cognitive (McClelland et al., 2000) and reading and vocabulary skills (Howse et al., 2003). *Sustained attention* while having ongoing distractions has been found to be positively associated with preschooler's cardinal principle knowledge, digit recognition and verbal counting (Brueggemann & Gable, 2018). Hence, *working memory*, *vocabulary* and *sustained attention* skills will be used as predictors of mathematical learning trajectories.

**Current methods used in early mathematical learning research**

The implementation of longitudinal methods is essential for obtaining an accurate understanding of early mathematical learning and change over time (Grammer, Coffman, Ornstein & Morrison, 2013). The cross-sectional and longitudinal studies that are available (e.g. Landerl, 2013; Manolitsis et al., 2013), have usually taken a traditional analytical variable-centered approach that suggest the relation between variables can be applied to all learners in the same way. For instance, Landerl (2013) found that numerical processing (i.e., processing digits and numerical sets) was evident as early as Grade 2 for both typically developing children and children with persistent arithmetic problems. Meanwhile, Manolitsis et al. (2013) showed that parents' teaching of literacy skills predicted reading fluency through the effects of letter knowledge and phonological awareness and parents' teaching of numeracy skills predicted mathematics fluency through the effects of verbal counting. These person-centred methods limit researchers' ability to deal with heterogeneity within and between individuals (Hickendorff et al., 2017) as they show average trends at the group level, but do not deal with the nuance of individual variation. These methods have limited our understanding of mathematical development and are not adequate to explore for example, how contextual variations (i.e. the home numeracy environment) may affect learning and development between groups of individuals differently (Lindblom-Ylänne, Parpala & Postareff, 2015). The current study will consider learner subgroups and how children's mathematical learning pathways change over time by tracking children from preschool to the first year of formal education and utilising a person-centered longitudinal research design.

The person-centered approaches that have previously been used (i.e., cluster analysis) have shown different profiles of skills regarding children's mathematical skills. In a study with kindergarten children in the U.S.A. ($M_{age}$ = 5.8 years), Jordan, Kaplan, Nabors Oláh & Locuniak (2006) showed that low SES children performed significantly worse than middle SES children at the end of kindergarten on most number sense measures (e.g., enumeration, counting sequence etc.). However, both groups of children progressed at around the same rate with the exception of story problems with low SES children achieved slower growth rates. Studies investigating older children's conceptual understanding of mathematics have also found children with different profiles of skills. Canobi (2005; Study one 7-9 years, Study two 5-7-year olds) demonstrated that different groups of children held different conceptions of the inverse relation between addition and subtraction. Gilmore and Bryant (2006; 6-8-year olds) found different subgroups of children regarding the relationship between their conceptual understanding of arithmetic and procedural arithmetic skills. Overall, these studies demonstrate that children may take different routes to understand and solve numeracy problems. These findings also emphasise the importance of considering the variety of ways in which children develop an understanding of mathematics. However, these studies are with older primary school aged children and therefore they do not reveal developmental pathways to early numeracy learning. Neither do these studies consider other factors that may be associated with subgroup membership. The current study will use a Latent Transition Analysis (LTA) to investigate growth in basic numeracy skills over time, allowing for the examination of transitions and the order in which numeracy skills develop for different subgroups of children."

## Current study

As highlighted above children's early numerical development includes understanding of multiple components (i.e. digit recognition, mapping magnitude representations, cardinal principle knowledge and order processing), and the development of these may be influenced by domain-general (i.e. working memory, vocabulary and sustained attention skills) and non-cognitive factors (i.e. SES and HNE). However, as of yet, no study has defined subgroups and developmental pathways of numeracy learning based on these fundamental specific numerical skills. Hence, this study aims to identify subgroups and developmental pathways based on these four fundamental skills. This study will also aim to identify whether predictors such as, working memory, language, sustained attention skills, SES and HNE are related to change in the developmental pathways using an LTA and regression analyses. Hence, the specific numerical predictors of mathematical development will be used to identify the subgroups of children and domain-general and non-cognitive factors will be used to predict developmental pathway membership. This study will lead to a deeper understanding of the order in which these numeracy skills develop for different subgroups and the predictors of developmental stability or change.

# Method

The research procedures were approved by [BLINDED] University Research Ethics Committee before the study commenced. Only children whose parents/guardians returned a signed consent form participated in the study. Children provided assent before the researcher began to administer the tasks.

## Participants

152 parents agreed to complete the Preschool Home Maths Questionnaire (PHMQ) at Time 1 and consented for their child's participation in the study. Children were assessed at three time points over an 8-month period from preschool to primary school, comprising two

time points in preschool. The majority of children in [BLINDED] change classes between preschool and primary one. There were 7 preschools involved in data collection at T1 and T2 and 14 schools involved in data collection at T3 with multiple classrooms and teachers. From the 152 dyads recruited at T1, a total of 136 parents (124 mothers and 12 fathers) completed the PHMQ. A total of 128 children ($M_{age}$ = 4 years; $SD_{age}$ = 3.3 months; age range 43 to 54 months; 70 female) completed the tasks at T1 and T2. All children that took part in T1 also took part in T2, children remained in the same classes with the same teachers between T1 and T2, there was no attrition between these points. Of these, 118 children (65 female) completed the tasks at T3. The 128 participants' data were used in the LTA. There was a 22.4% loss rate from T1 to T3 (i.e., 152 dyads originally recruited with 34 dyads lost to attrition in follow-up between T2 to T3, leaving 118 dyads with available data at T3). The LTA will estimate where children start (T1, early Spring term of pre-school in February) giving their initial group profile and then provide the same information for Time 2 (T2, late Spring term of pre-school in May) and after the children's transition from pre-school to primary school education (T3, Autumn term of Primary one in October; see Figure 4 for a visual of the transition from preschool to school).

**Procedures**

Parents completed the PHMQ and returned it to the preschool teacher to be collected by the researcher at T1. At T1 and T3 children completed tasks over three 20-minute sessions. The tasks measured 1) specific numerical skills i.e. cardinal principle knowledge, digit recognition, order processing and symbol-quantity mapping, 2) domain general skills i.e. verbal working memory and sustained attention, 3) receptive vocabulary and 4) general mathematical achievement. At T2 children completed two 20-minute sessions; receptive vocabulary and mathematical achievement measures were not included at T2.

## Measures

### ***Socio-Economic Status***

Two proxies of SES that could influence mathematical development were used; 1) parental education and 2) occupational measures classified by using The National Statistics Socioeconomic Classification (NS-SEC; Rose & Pevalin, 2010).

### ***Preschool Home Maths Questionnaire (PHMQ)***

The Preschool Home Maths Questionnaire (PHMQ; AUTHORS, 2021, AUTHORS, 2017) is a newly developed questionnaire that assesses different areas of the HNE (e.g., parent expectation, literacy, counting ability, parent-child teaching methods, frequency of numeracy activities etc.). The frequency of numeracy activities scale is used within this study as a predictor of mathematical development (Appendix A). The five subscales were labelled as follows; parent - child interactions, computer maths games, TV programmes, shape and counting – and comprised 29 items. Although the confirmatory factor analysis demonstrated that the five-factor model was a suitable measurement model the sub-scales and scale (i.e. as an overall score) does not correlate with the mathematics specific skills (see Appendix C, Table 3). Due to the lack of correlation between mathematics specific skills and the frequency of numeracy activities subscales only an overall score of the frequency of numeracy activities scale will be used as a predictor of pathway membership. Cronbach's alpha for the total scale was .89. Cronbach's alpha for the subscales based on these factors were acceptable, ranging from .76 for the *counting* factor to .81 for both the *parent - child interactions* and *computer maths games* factors, thus the scale displays good internal reliability. A confirmatory factor analysis was used to quantitatively assess the quality of the five-factor structure of the frequency of numeracy activities scale. Taking into consideration all criteria for assessing goodness of fit (CFI = 0.83, TLI = 0.81, RMSEA = 0.07, SRMR = 0.072; see AUTHOR, 2021 for more details), the five-factor model (with sub-factors of parent-child interaction, computer maths games, TV programmes, shape and counting) was deemed a suitable measurement model. The five subscales demonstrate a comprehensive breakdown of numeracy related activities occurring in the home.

***Digit Recognition***

Arabic digits (font Arial; size 200; landscape) were presented to children who were asked, “What number is this?”. The instruction could be repeated once, if needed. There were 2 blocks with 6 numbers in each. The first block contained single digit numbers (in this order 3, 2, 5, 8, 7, 9) and the second block contained numbers between 10 to 20 (in this order 12, 14, 11, 16, 20, 18). The researcher stopped the test when a child made a mistake with four or more numbers in one block. A proportion score out of 12 was calculated for each participant, including those who completed only the first block. The Cronbach’s alpha for the digit naming task was 0.91 in Batchelor and colleagues (2015) study.

***Symbol-quantity Mapping – Cross-notation Comparison task***

Following Batchelor and colleagues (2015), children were presented with two numerosities and two characters, a duck and a frog, and instructed to choose which of the two characters had the most balls. One of the characters’ balls were presented as a dot array and the other characters’ balls were presented as a number word. Each child completed 12 trials that were counterbalanced as the researcher presented the first numerosity as a verbal number word for 6 trials and for the other 6 trials the first numerosity was non-symbolical dot arrays on a laminated card (18cm x 12cm). The numerosities used ranged between one and fifteen. To aid understanding of the verbal number word trials, the child was presented with a picture of a box on a card (i.e. the character had put their balls in a box). Both of the presented cards stayed on the table next to each of the characters (see Figure 1) until a response was given by the child. Children responded by pointing to and/or naming the character. Children were told not to count the dot arrays. If a child made any visible counting acts (e.g. counting utterances) the child's response was marked down as incorrect as this task required children to directly assess mapping between magnitude representations without counting. To familiarise the child with the instructions two practice trials were administered that were non-symbolic (dot array vs. dot array). Figure 1 (a) is a practice trial example of a non-symbolic (dot comparison) and Figure 1 (b) is an example of a cross-notation comparison trial. For each correct response children were awarded one point, giving a maximum score of 12. A proportion score was calculated for each participant. The Cronbach's alpha for the cross-notation comparison task was 0.64 in Batchelor and colleagues (2015) study.

***Cardinal Principle Knowledge – Give-N task***

The Give-N task was adapted from Wynn (1990) and was presented in line with previous studies (e.g. Batchelor et al., 2015; Condry & Spelke, 2008). The researcher placed 18 plastic counters (each 1.5 cm in diameter) in a cluster in front of the child (see Figure 2). The child was asked to give a colourful puppet called Fluffy (32 cm high) a given number *[n]* of the counters by placing the counters on a yellow plastic plate (25 cm in diameter). For example, "Can you give three counters to Fluffy?" Following the child's response, the researcher asked; "Is that *[n]*?" If the child responded with "yes," the researcher proceeded with the next trial. If "no," the researcher repeated the original request. There were five different colours for the five different numbers asked (e.g. 18 white, 18 green counters etc.). The five numbers used in this task were 3, 4, 6, 11 and 15. Each number was requested up to three times and for each correct response children were awarded one point. A proportion score out of 15 was calculated for each participant. Batchelor and colleagues (2015) used a similar task which had a Cronbach's alpha of 0.76 for the Give-N task.

***Order processing***

Children were presented with three number cards (12.5cm x 9 cm) horizontally as Arabic digits (font Calibri, size 140, portrait). The cards were presented out of order and children were asked "Can you put the numbers in the right order from the smallest number to the biggest number?". Trials were four *consecutive* (e.g. 8, 7, 9), *gaps of 2* (e.g. 5, 9, 7), and *gaps of 3* (e.g. 1, 7, 4) numbers counterbalanced. There were 12 one-digit trials that were placed from left to right by the researcher. Before the 12 test trials commenced 4 practice trials were given. Only during the practice trials if the order of the numbers were placed in the incorrect order by the child after the second attempt the researcher stated, "Good try, but if we put the numbers like this [arrange into order] then they are smallest to biggest". This instruction was not given in the experimental trials. No time limit was put in place for children to move the number cards. For each correct response children were awarded one point, giving a maximum score of 12 and a proportion score was calculated for each participant.

***Receptive Vocabulary – British Picture Vocabulary Scale***

The British Picture Vocabulary Scale – Third Edition (BPVS-III; Dunn, Dunn, Styles & Sewell, 2009) is a standardised non-reading assessment of receptive vocabulary. The researcher states a word and the child responds by selecting a picture from four options that best illustrated the word's meaning. There are 14 sets giving a maximum score of 168. The BPVS-III was normed on 3278 children with and without disabilities aged 3 to 16 years and has an internal reliability of r = 0.91 and criterion validity with the Wechsler Intelligence Scale for Children (2005) of r = 0.76 (Dunn & Dunn, 2009; Hannant, 2018).

***Verbal Working Memory – Animal Recall task.***

Working memory was measured using an adapted version of an animal recall task (McCormack, Simms, McGourty & Beckers, 2013). The task involved inter-leaved presentation of animal cards and smiley faces. As the cards were presented, the children were asked to name the colours of the smiley faces and at the end of each set they were asked to recall the animals in the correct order once the researcher asks, "What animal(s) did you see?". A total of 22 animals were used as stimuli, all animals were recognisable by children aged 3-5 years old. Children initially received 4 practice trials that were repeated if necessary. The practice trials consisted of two sets of one-animal and two-animal trials, with subsequent levels involving four sets of one- through five-animal trials, thus five levels of increasing difficulty. If all the animals in one of the four trials in a level were recalled, even in the incorrect order, children moved to the next level. The task was terminated when the child failed to recall all the animals in any of the four trials at a level. No animal was repeated within a level and no animal was repeated more than 3 times throughout the whole task. For each correct response, which was classified as an animal recalled in the correct position, children were awarded one point and the accuracy score was used in the analysis. The maximum score was 60, however, no child scored above 20 at all three-time points.

***Auditory Sustained Attention – Continuous Performance Test – Preschool (ACPT-P)***

The Auditory Continuous Performance Test-Preschool (ACPT-P) developed by Mahone, Pillion and Hiemenz (2001) provided the basis for the development of an adapted computerised Continuous Performance Test (CPT) used in this study. The task included 44 trials, each consisting of 22 non-targets and 22 targets. Two familiar environmental sounds (dog barking and cat meowing) were used as stimuli. The stimuli were presented for 1 second and children had 4 seconds to respond. The children were seated in front of a laptop and heard both a dog barking and cat meowing at different intervals. Participants were required to press the spacebar for the target sound (dog barking) and to inhibit their response by abstaining from pushing the spacebar for the non-target (cat meowing). Children were given a familiarisation phase, followed by a practice phase, that aimed to confirm the child's comprehension of the instructions. All participants were able to successfully complete the practice trials before continuing onto the test phase.

The accuracy score, total omission, total commission and response rate were calculated. The first three scores had a maximum score of 22 and the response rate ranged from 0-4 seconds. An inverse efficiency score (IES) was used to enable the combination of speed and error metrics (Townsend & Ashby, 1978). IES is calculated by taking the participants average correct response time (RT) of the condition divided by 1 take away the proportion of errors (PE; i.e. IES = RT / (1 – PE); Bruyer & Brysbaert, 2011).

**Data Analyses**

Descriptive statistics and regression analyses were conducted using SPSS. The British Ability Scale (BAS-II) Early Number Concepts (Elliot, Smith, & McCulloch, 1996) measure of general mathematical achievement and a spontaneous focus on numerosity picture task (SFON; Batchelor et al., 2015) were also administered. However, BAS was not included in the data analysis due to our focus on componential numerical skills and after preliminary analysis SFON was not included in any of the analyses due to a lack of variability in scores. The Latent Transition Analyses (LTA) were completed in Mplus Version 1.5. An LTA maximises *similarity within* the subgroup and *difference between* the subgroups (Lanza & Cooper, 2016). This is different from a variable-centred approach were the emphasis is on identifying relations between variables, and it is assumed that these relations apply across all people. Prior to conducting the LTA, a series of Latent Profile Analysis (LPA) were conducted (Lazarfeld & Henry, 1968; Muthén, 2001) this is a simpler cross-sectional model (i.e. LPA or latent class analysis (LCA)) than the more complex longitudinal model (i.e. LTA) and allows the researcher to investigate each time point separately. LPA confirms the extent that the extracted latent profiles can be replicated at each individual time point (Kam, Morin, Meyer & Topolnytsky, 2016). The transitional component of an LTA reflects changes in subgroup membership over time, demonstrating potential non-linear learning pathways (Hickendorff et al., 2017). LTA estimates where children start (T1, preschool) giving their initial subgroup and then provides the same information after the children's transition from preschool to primary school education (T3, primary one). Through LTA children's precise *learning pathways* during the transition from preschool to primary education are described (Fryer, 2017).

The LTA had four indicators of specific numerical skills based on each time point. To deal with the missing data full information maximum likelihood (FIML) was used. The criteria for making the decision on how many subgroups there are depends on the theoretical and practical meaning to the extracted profiles (Marsh, Lüdtke, Trautwein & Morin, 2009) and the statistical adequacy of the solution. When the best model was identified through the LTA a framework was used to avoid a measurement parameter shift problem once predictors are added into the model, known as the three-step specification. The three-step method estimates the effects of auxiliary variables (i.e. covariates (or predictors) and distal outcome) in mixture models (Asparouhov & Muthén, 2013; Vermunt, 2010) by ensuring that the measurement of the latent profile variable (i.e. in this study the mathematics profile) is not affected by the inclusion of predictors by fixing the measurement parameters of the latent profile variable with predictors at values from the unconditional latent class model (i.e. the final retained model).

The second step is assigning individuals to latent profiles using the logits of classification probabilities (i.e. the average latent class probabilities for most likely latent class membership). To do this the model was run again with a "savedata" command which requests Mplus to create a new dataset with "cprobabilities", or classification probabilities. The majority of participants had high classification probabilities (i.e. 80%+ cprobabilities for 88% of the sample) in the pathways. Thus, it was deemed appropriate to complete a regression analysis on the LTA pathways by reading in these data into the next model. A pathways variable was created that matched to each participants' pathway membership.

Finally, the mixture model is estimated with measurement parameters that are fixed at values that account for the measurement error in the class assignment (see Asparouhov &

Muthén, 2014; Nylund-Gibson, Grimm, Quirk & Furlong, 2014; Vermunt, 2010 for more information on three-step approaches). After the three-step method, predictors can be included in the model in the traditional manner and the predictors should not impact the model. This approach is an acceptable alternative method to the three-step method (Asparouhov & Muthén, 2014) that provided a framework for avoiding a measurement parameter shift problem once predictors were added into the model.

Some simulation studies exist and have examined sample size for LCA models (e.g. Dziak, Lanza & Tan, 2014; Wurpts & Geiser, 2014). These studies argue that small sample size may be overcome by including important covariates (i.e. in this study working memory, attention, SES etc.), higher quality indicators (i.e. those indicators with strong relation to the latent variable (i.e. in this study the four basic numeracy skills)) and/or having many indicators (i.e. in this study the four basic numeracy skills) but that generally larger sample sizes (e.g. approx. 500) are preferable. However, the sample sizes within these papers are recommendations related to LCA- cross-sectional analysis. Whereas, the approach used within this study, LTA, is the longitudinal extension of the LCA model. The longitudinal nature of this study strengthens the appropriateness of using LTA with the current sample size, as the 128 participants responded 3 times giving a total number of 374 data sources. Therefore, LTA was deemed appropriate for the sample size of this study. Furthermore, to establish "factorial" measurement invariance initial LPA models was fitted to wave one data and the same model structure was replicated across waves 2 and 3. These replications at wave 2 and 3 clearly strengthen the appropriateness and relevance of the LPA model, prior to conducting the LTA. The Mplus code has been provided (see Appendix B) to enhance the visibility of the analysis.

This study considers a variety of demographic characteristics (i.e. gender, age, SES), as well as predictors associated with multiple components of the HNE measured with parents (i.e. frequency of numeracy activities and number games checklist; two dimensions from the PHMQ), and domain-general child measures (i.e. working memory, sustained attention and vocabulary). A multivariate multinomial logistic regression analysis was used to understand the potential predictors of pathway membership. Blocks of predictors were entered into the model in a forward stepwise manner. This was a statistically driven model, due to the results of a multinomial logistic regression that was completed to understand the bivariate associations between pathways membership and the individual predictor variables. Thus, predictors were selected for this final adjusted model if $p < .05$ for any association between pathway membership and the given predictor in the bivariate analyses. T-tests were conducted to compare receptive vocabulary, working memory, sustained attention and frequency of numeracy activities between the different developmental pathways.

## Results

Table 1 shows the socioeconomic status classification and educational attainment breakdown. Table 2 demonstrates the descriptive statistics for each measure across the three-time points. There was significant change in all measures over time.

### Determining the latent transition analysis model

A LTA was completed and the model fit indices for 2 to 5-profiles solution were extracted to evaluate the best fitting model (see Table 3 and Appendix C, Table 4 for the number of participants in each profile based on 128 participants data available for the LTA). The three-information criterion (IC) indices are selection criterion, where lower values reveal the preferred model; Akaike's information criterion (AIC), the Bayesian information criterion (BIC) and the sample size - adjusted BIC (aBIC). The model with the smallest IC values was selected. Following previous research (Nylund-Gibson, Grimm, Quirk & Furlong, 2014) the elbow of the BIC value (the last large decrease in the BIC value) was used as a guide (Fryer, 2017) and a three-profile model was supported. This was also supported by the entropy value settling at a relatively higher amount (S = 0.951) for this solution, suggesting good separation of the profiles (Nylund-Gibson et al., 2014; Fryer, 2017).

This finding suggests that children could be meaningfully grouped into three separate subgroups; at each time point there was a low-scoring, intermediate-scoring and advanced-scoring subgroup. Membership of these three groups may not be the same at each time point (i.e. children can move between subgroups over time). To examine how children's mathematical skills, develop and transition over time we examined the latent transition *pathways*.

**Latent transition pathways**

The results reveal that only 7 (i.e. 26%) of the possible 27 pathways had been taken by at least one child. Two pathways were followed by only 1 participant each (<5% of participants). Previous literature suggests that pathways should not be analysed further if less than 5% of participants travelled a specific pathway (Schneider & Hardy, 2013). Therefore, these pathways will not be discussed in further detail. The remaining 5 pathways describe the development of 126 (98.4%) of the population.

**Characteristics of the pathways**

There were three consistent number skills pathways and two pathways in which children shifted from one profile to another upon school entry (see Figure 4 for visual). Note, there was no relationship between school attended and pathway membership at either T1/T2 ($x^2$ (24, N = 126) = 26.53, p = .24) or T3 ($x^2$ (52, N = 126) = 53.35, p = .24). Pathway 1 was named the *consistently low number skills pathway* as this subgroup scored low on all of the number skill tasks over time, consistently scoring below and at chance on the mapping task (see Figure 3 for breakdown of pathway characteristics over time). Pathway 2 was termed the *low to intermediate number skills shifting pathway (at school-entry)* as this subgroup was in the lowest group during preschool (T1 and T2) but at school-entry (T3) there is a shift in their level of understanding of these four skills and they moved into the intermediate group. Over time (T2 to T3) these children continue to develop and understand cardinal principle knowledge and score above chance on mapping skills. These children only marginally continue to progress in their digit recognition and ordering skills. This subgroup scored higher on digit recognition and ordering skills than the Pathway 1 subgroup at T3.

Pathway 3 was called *consistently intermediate number skills pathway* as this subgroup were consistently in the average group. Examining these children's scores on the four numerical skills tasks suggests that this subgroup already had an understanding of cardinal principle knowledge at T1. The children scored higher on this task than the other numerical tasks, this might suggest that cardinal principle knowledge is the first number skill that children gain an understanding of from these four number skills. This subgroup scored just above chance in the mapping task. Further, the subgroup comprehended digit recognition skills and therefore, began to develop order processing skills. Children's performance was similar at T2. By T3 children showed improvement in their mapping and ordering skills.

Pathway 4 was termed *intermediate to advanced number skills shifting pathway (at school-entry)* as this subgroup were in the intermediate group at T1 and T2 but moved into the advanced group at T3. At T3 the subgroup scored high in all four number skill tasks. Pathway 5 was named *consistently advanced number skills pathway* since children scored high at all time points with the only notable improvement being in ordering skills.

Figure 4 displays children's transition pathways. Pathways 2 and 4 are the two pathways where children move up a profile. Fifty-seven (44.5%) children make a substantial change in their number skills development between T2 and T3, during the transition between preschool and primary school. Whereas, 69 (53.9%) children remained in their profile (i.e. Pathways 1, 3 and 5) over the three time points.

**Predictors of pathway membership**

To understand the potential predictors associated with pathway membership (i.e., the five pathways) a multivariate multinomial logistic regression analysis was used. The predictors used were cognitive (i.e., working memory, vocabulary and sustained attention) or non-cognitive factors (i.e., gender, SES and HNE). Pathway 5 was used as the reference class as this was the highest scoring basic numerical skill pathway and had the largest number of participants. Gender, sustained attention, frequency of numeracy activities scale and number games checklist were not significant predictors in the bivariate model.

A multinomial logistic regression was completed to understand the bivariate associations between pathways membership and the predictor variables. By first considering the bivariate associations the overall relationships are assessed between latent pathways and predictors without collinearity concerns. Furthermore, the differences between the unadjusted and adjusted estimates can be explored. The unadjusted odds ratio is obtained by only studying the effect of one predictor variable, whereas when more than one predictor is considered an adjusted odds ratio is created that takes into account the effect due to all the additional variables included in the analysis. The adjusted associations were considered through a multivariate multinomial logistic regression analysis after the unadjusted multinomial logistic regression. Predictors were selected for this final adjusted model if $p < 0.05$ for any association between pathway membership and the given predictor in the bivariate analyses (see Table 5 for the adjusted associations and the supplementary information, Appendix C, Table 1 for the bivariate associations).

**Adjusted associations between pathway membership and predictors**

Blocks of predictors were entered into the model in a forward stepwise manner: 1) demographic characteristics (i.e. age, SES and higher education qualification (HEQ)) and 2) child measures (i.e. working memory and vocabulary; see Table 4 for means across pathways).

In the final adjusted model (see Table 5) compared to the consistently advanced number skills pathway (Pathway 5), membership in the consistently low number skills pathway (Pathway 1) and the low to intermediate number skills shifting pathway (at school-entry) (Pathway 2) were significantly associated with age at T1 (OR = 0.61, $p < 0.05$, OR = 0.81, $p < 0.05$, respectively). The odds ratios demonstrate that as age increases children are less likely to be in Pathways 1 and 2 than Pathway 5. In short, for an additional month alive, a child's odds of being in Pathway 1 decreased by 39% (compared to Pathway 5), and by 19% for Pathway 2 (compared to 5).

Membership in Pathway 1 (compared to 5) was significantly associated with receptive vocabulary (OR = 0.89, $p < 0.001$) but not working memory. The odds ratios demonstrate that as receptive vocabulary scores increase by one-unit children are less likely to be in Pathway 1 than 5. In short, for a one-unit increase in receptive vocabulary, a child's odds of being in Pathway 1 decreased by 11% (compared to 5). Membership in Pathway 2 (compared to 5) was significantly associated with receptive vocabulary (OR = 0.93, $p < 0.01$) and working memory (OR = 0.76, $p < 0.05$). The odds ratios demonstrate that as receptive vocabulary and working memory scores increase by one-unit children are less likely to be in Pathway 2 than 5. For a one-unit increase in receptive vocabulary, a child's odds of being in Pathway 2 decreased by 7% (compared to 5) and for a one-unit increase in working memory, a child's odds of being in Pathway 2 decreased by 24% (compared to 5). There was no significant difference between the *consistently intermediate number skills pathway* (Pathway 3) and Pathway 5 with receptive vocabulary or working memory. There was a significant difference between the *intermediate to advanced number skills shifting pathway (at school-entry)* (Pathway 4) and Pathway 5 with working memory (OR = 0.79, $p < 0.01$) but no significant difference with receptive vocabulary. The odds ratio illustrates that as working memory scores increase children are less likely to be in Pathway 4 than 5. For a one-unit increase in working memory, a child's odds of being in Pathway 4 is 21% lower than Pathway 5.

SES was not significant in the final adjusted model (see supplementary information, Appendix C, Table 2 for frequencies of demographic characteristics per pathway). With regards to parent's HEQ middle secondary school level qualifications (i.e. General Certificate of Secondary Education (GCSE) in United Kingdom (UK)) level was used as the reference category. Compared to pathway 5, membership in the Pathway 1 was significantly associated with the HEQ degree and no qualification (OR = 0.01, $p < 0.05$; OR = 0.02, $p < 0.05$, respectively). The odds ratios demonstrate that if parent's highest educational qualification is degree or no qualification children are less likely to be in the *consistent* Pathway 1 than in Pathway 5. Between the Pathway 2 and 5 all HEQ levels were significantly associated (higher secondary school level qualifications (i.e. A-level in the UK) OR = 0.05, $p < 0.01$; and higher education qualifications such as degree OR = 0.05, $p < 0.01$; masters OR = 0.02, $p < 0.001$; PhD OR = 0.05, $p < 0.05$; as well as no qualifications OR = 0.22, $p < 0.01$).

Compared to Pathway 5, membership in the *consistently intermediate number skills pathway* (Pathway 3) was significantly associated with the HEQ A-levels, degree and masters (OR = 0.06, $p < 0.05$; OR = 0.01, $p < 0.001$; OR = 0.03, $p < 0.01$, respectively). The odds ratios demonstrate that if parent's HEQ is A-levels, degree and master's children are less likely to be in Pathways 3 than 5. Finally, compared to Pathway 5, the *intermediate to advanced number skills shifting pathway* (at school-entry) (Pathway 4) was significantly associated with the HEQ A-levels, degree, masters and no qualifications (OR = 0.05, $p < 0.05$; OR = 0.08, $p < 0.05$, OR = 0.04, $p < 0.05$; OR = 0.04, $p < 0.05$, respectively). However, based on the zero frequencies, or near zero frequencies, among both SES and HEQ categories, results should be taken with caution.

**T-test analyses**

The above analyses compared children in each of the pathways with Pathway 5 (consistently advanced number skills). However, of particular interest is the difference between the consistent and transition pathways (i.e. children who had similar numerical skills at T1 and T2 but some of these children showed greater improvement at T3. Therefore, we conducted independent-samples t-tests to compare receptive vocabulary, working memory, attention and frequency of numeracy activities for those children in the consistent pathway versus those in the transition pathway (i.e. comparing Pathway 1 to Pathway 2 as well as comparing Pathway 3 to Pathway 4). Results demonstrated no significant difference between both consistent and transition pathways on working memory, attention and frequency of numeracy activities. However, there was a significant difference between the children on Pathway 1 ($M = 41.56$, $SD =10.48$) and the children on Pathway 2 ($M = 50.68$, $SD = 13.26$); $t (45) = -2.39$, $p = .02$ (two-tailed) for receptive vocabulary scores.

## Discussion

The current study reinforces that children vary substantially in their level of number knowledge prior to school-entry, consistent with previous research that proposes that mathematics knowledge begins to develop at a young age to varying degrees (Manolitsis et al., 2013). Further, this study strengthens the idea that mathematical cognition is not unitary but consists of many components that develop at different rates (Dowker, 2008). Due to the use of four concurrent measures of mathematics-specific skills and the longitudinal nature of this study these data provide significant insight into mathematical development during preschool.

Five developmental pathways were discovered that describe children's precise *learning pathways*; three consistent pathways, where children stayed in their subgroup across all three time points and thus progressed at a steady rate, and two school-entry transition pathways where children displayed a more rapid shift to the subsequent, more advanced, learner profile upon school-entry. Interestingly, no pathway skipped a profile (i.e. no child skipped from a low number skills profile to the advanced number skills profile) but instead children systematically transitioned through each subsequent profile (i.e. low number skills to the intermediate number skills profile). The two transitions between profiles were observed at school-entry, demonstrating that many children make substantial learning gains once they enter school. During school children are prompted to make connections between new information and prior, or existing knowledge, by practicing and recalling new information in different ways (e.g. revision or questions) leading to the consolidation of their learning material (Howard-Jones et al., 2018). Therefore, as the transitions between profiles occurs at school-entry perhaps this is due to children consolidating their mathematical skills.

It is important to note that although 69 children (53.9%) remained in their learner profile over time their mathematical skills were still developing. However, substantial developmental change was observed in the performance of the 57 children (44.5%) in the school-entry transition pathways; exemplified by these children transitioning between profiles. Importantly, there was no pathway that demonstrated a shift to a less advanced profile due to children making substantially slower progress. This is similar to Schneider and Hardy (2013) who completed a LTA and found that no pathway demonstrated a decrease in conceptual knowledge associated with the scientific topic of floating and sinking throughout an intervention study.

There was some evidence that the four mathematics specific skills developed in a consistent way. When comparing pathways, in particular consistent pathways (e.g. comparing Pathway 1 with 3 and Pathway 3 with 5 etc.), children develop cardinal principle knowledge and symbol-quantity mapping skills that involved verbal number words but not written Arabic digits before developing digit recognition skills and order processing abilities that require the use of symbolic information (i.e. Arabic digits). This is consistent with previous research suggesting that children form connections between number words and quantities before they form connections between Arabic digits and quantities (Lira, Carver, Douglas & LeFevre, 2017); emphasising the importance of verbal number knowledge as a foundation for later mathematical development.

For those in Pathway 1 the rates of growth in these mathematics specific skills is very slow. Cardinality is the first skill to develop which could suggest that the acquisition of cardinality allows for changes to children's understanding of number (Slusser & Sarnecka, 2011). Pathway 2 over time (from T2 to T3) these children continue to develop and understand cardinal principle knowledge and score above chance on mapping skills. These children only marginally continue to progress in their digit recognition and ordering skills. Therefore, it could be assumed that these children potentially understand the concept and meaning of number words however, they are unable to order numbers as they do not recognise digits fluently. There is development in digit recognition before the understanding of ordering which is plausible. Cardinality and symbol-quantity mapping develop at the same time with the jump in cardinal skills possibly accelerating the understanding of magnitude representations (see Pathway 2, T3), or vice versa. It could be that these skills developed over time by influencing and reinforcing one another across development similar to the mutual development theory wherein principles and experiences develop together (Sarnecka & Carey, 2008). This is in line with previous work which found that some children could map between number words and quantities but were not cardinal principle knowers, while other children were cardinal principle knowers but could not map between number words and quantities (Batchelor et al., 2015). Understanding what impacts these developmental pathways is essential.

**Predictors of pathway membership**

Findings showed that younger preschool children are less likely to develop their number skills over time. These younger children (who were on average 3.56 (Pathway 1) and 3.61 (Pathway 2) years old) in the low number skills *pathways* have not developed cardinal principle knowledge yet, which would typically develop around 3 years of age (Bermejo, 1996). Previous research indicates that the first step to cardinal principle knowledge occurs when children learn the first few number words (Gelman & Gallistel, 1978) between 2 to 3 years of age, progressing over the next two years by gradually understanding larger number words (Le Corre & Carey, 2007; Wynn, 1990).

Results demonstrate that as children score higher in receptive vocabulary, they are more likely to be in the consistently advanced number skills pathway than the lower number skills pathways. Thus, findings are consistent with growingly popular ethnomathematical literature that indicate that language may impact the learning of mathematics (Kim et al., 2012). Children in the lowest number skills pathways were more likely to have low number skills and low receptive vocabulary scores. This, along with slow rates of growth over time for those children in the low number skills pathway may have profound negative outcomes as literature shows that children who enter preschool with low performance in basic number skills and low levels of early literacy skills stay behind their peers throughout later school years (Jordan, Kaplan, Locuniak & Ramineni, 2007; Jordan et al., 2006).

In the present study, children in the advanced number skills pathway (Pathway 5) had higher working memory than children in the other four lower number skill pathways. This is consistent with previous literature and meta-analyses that proposes that those with higher working memory capacity performed better on difficult mathematics problems (Peng, et al., 2016). Additionally, there was an association between working memory scores and the school-entry transition pathways. Working memory is one of the most fundamental executive functions that is present early in life and shows rapid development throughout preschool (e.g. Bull et al., 2008). As stated earlier, most longitudinal evidence between cognitive and mathematics skills focus on school-age children and adults. Evidence does indicate this association is present before school-entry, although research is limited (e.g. Bull et al., 2008). Bull and colleagues (2008) found a significant yet small contribution was made by verbal working memory (as measured by the digit backwards task) to mathematical skills at school-entry. Therefore, the current findings confirm previous research as there is a significant relation between verbal working memory and the school-entry transition pathways and provides new evidence that working memory skills could enable the shift in mathematical learning pathways at school-entry.

**Consistent versus shifting pathways**

The results suggest that receptive vocabulary skills have an association with some developmental pathways (i.e. consistent versus shifting pathways) as children on Pathway 2 had significantly higher receptive vocabulary scores than children on Pathway 1. Specifically, those children on Pathway 2 have higher receptive vocabulary scores. This confirms the claim that mathematics is a language-dependent skill (Kim et al., 2012) as vocabulary knowledge is involved when counting and solving numeracy problems (Jordan et al., 2003). These findings suggest that supporting vocabulary knowledge, particularly for those children with lower numerical skills, may increase children's number knowledge.

**Limitations and future recommendations**

Sustained attention at T1 was not a significant predictor in the bivariate model and hence not included in the adjusted multivariate model. In preschool education the development of sustained attention is not completely understood and there is a somewhat scarce in the research literature base (Guy, Rogers & Cornish, 2013) possibly due to the fact that sustained attention is difficult to measure in preschoolers. Brueggemann et al. (2018) found a positive association between sustained attention and cardinal principle knowledge, digit recognition and verbal counting using a *visual* sustained attention task (i.e. Track-It task) whereas the current study uses an *auditory* sustained attention task. More research is necessary to refine the Continuous Performance Test (CPT) suitable for preschool children. CPTs are mainly used with school-aged children and adults rather than with preschoolers due to difficulties associated with task length, number of distractors and short inter-stimulus intervals (Guy et al., 2013; Mahone et al., 2001). Therefore, perhaps the differences in findings are due to the different domains targeted within the studies (i.e. visual and auditory). Furthermore, it is worth mentioning that attention skills increase while children are engaged in academic studies (Duncan et al., 2007) therefore, as preschool is not formal education this is perhaps further reason for sustained attention not being a predictor of any pathway membership as attention skills are not yet developed.

HEQ categories were more likely to be significantly associated with pathway membership than SES categories consistent with previous literature which suggests parental education is the best predictor of academic achievement (Sammons et al., 2004). However, the SES and HEQ results should be taken with caution due to zero frequencies or *near to* zero frequencies among some SES and HEQ categories. It is recommended that in future research more data be collected to explore the causal effects of SES and HEQ on pathway membership.

The findings from this study indicated no statistically significant relations between parents' reported frequency of numeracy activities in the home with pathway membership at a bivariate level. There have been inconsistent results concerning the relation between home activities (measured via questionnaires) and children's number skills (e.g. Skwarchuk, Sowinski & LeFevre, 2014; LeFevre et al., 2009). An important reason for the discrepancy between results may involve the different ages of the children in these studies. Some research has found unique and positive associations between the HNE and mathematics skills with children between the ages of 4-years 10-months to 8-years (e.g. Kleemans et al., 2012 ($M_{age}$ = 6.1 years, age range = 5 to 7 years); Dearing et al., 2012 ($M_{age}$ = 6.72 years, SD = 0.34); Manolitsis et al., 2013 ($M_{age}$ = 64.32 months, SD = 3.23, age range = 5 to 6 years)). Whereas, Missall et al. (2015) in a study involving children aged 3 to 5 years 7-months ($M_{age}$ = 53.6 months) found no relation between mathematics-related activities in the home and a range of numeracy skills. In the current study children were aged between 4 years and 4 years 7-month, similar to that of Missall et al. (2015). The measure of *informal home numeracy practices* (number games exposure checklist) was also not significant in the bivariate model. The measure previously developed by Skwarchuk et al. (2014) was correlated significantly with later mathematics skills. Further, informal exposure to numerical board games predicted children's non-symbolic arithmetic (Skwarchuk et al., 2014). However, children in Skwarchuk et al. (2014) study were aged 5 years 3 months to 6 years 5 months ($M_{age}$ = 58 months). Hence, the younger age group included within this current study could also explain the results. Most countries in Europe, and across the globe, have a school starting age of 6 (Sharp, 2002). In the UK compulsory schooling starts at 5 years of age. In practice however, most children start Primary 1 at the beginning of the year in which they become five (Sharp, 2002). Hence, the children in this current UK sample are starting primary school earlier than children in many other countries, this could be a contributing factor to the inconsistent findings. Furthermore, parents are engaging in home numeracy activities within the current study but perhaps children are too young for this engagement to make a difference to their learning trajectories.

LTA was deemed appropriate for the sample size of this study as the 128 participants responded 3 times giving a total number of 374 data sources. Nevertheless, the primary limitation of this study was that this framework cannot consider the new three-step specification method due to sample size limitations (Vermunt, 2010; Asparouhov & Muthén, 2014). The three-step method deals with the measurement parameter shift problem by fixing the parameters estimated based on an unconditional model and in turn the predictors do not impact the nature of the model (Marsh et al., 2009; Morin et al., 2011; Ryoo, Wang, Swearer, Hull & Shi, 2018). Although an alternative method was utilised within this study that controlled for this movement, the recently developed three-step method has been demonstrated to be less biased, have lower mean squared error, and better confidence interval coverage than traditional methods. Thus, more research is necessary to ensure that the three-step method is generalisable (Ryoo et al., 2018). Furthermore, increased sample size may be desirable to utilise this three-step method even though the majority of participants had high class probabilities in the pathways.

In order to explore whether there could be school level effects (i.e., the school children attended) on pathway membership frequency data was explored. Chi-square tests found no association between school and pathway membership therefore, there is no evidence that school of attendance influenced pathway membership.

## Conclusion

Previous cross-sectional approaches have often failed to address what is associated with children's mathematical learning changes during school transitions. However, using a person-centered approach to investigate subtypes of individuals that exhibit similar patterns of characteristics has enabled us to identify precursor skills that contribute to more complex mathematical development over time. This study demonstrates that those children in the low number skill pathway (Pathway 1) have a lower rate of growth than children in more advanced pathways, possibly due to a lack of understanding in cardinality. This study enhances the previous research of Geary et al. (2018) in finding that cardinality is a critical part of foundational numeracy skill development. Future research should seek to confirm the role of cardinality understanding in driving further mathematical development in order to quantify the size of this effect. The discovery that children in the low number skill pathway (Pathway 1) have a lower rate of growth than children in higher number skill pathways supports Garon-Carrier et al. (2018) finding that children (i.e. aged 4 to 10 years) with low number knowledge persistently scoring lower than the other groups throughout the years. Understanding children's rates of growth is important as it allows educators to provide appropriate, targeted support and also contributes to our theoretical understanding of the development of number skills. Overall, the results indicate that many factors (i.e. age, vocabulary and working memory) contribute to mathematical skill development (i.e. pathway membership) over time and there is no one factor solely driving mathematical development. These findings highlighted the potential importance of language and working memory abilities on mathematical skills development in 3-5 year-olds.

## References

Abedi, J., & Lord, C. (2001). The language factor in mathematics tests. Applied measurement in education, 14(3), 219-234. https://doi.org/10.1207/S15324818AME1403_2

Asparouhov, T., & Muthén, B. (2014). Auxiliary variables in mixture modeling: Three-step approaches using M plus. Structural Equation Modeling: A Multidisciplinary Journal, 21(3), 329-341. https://doi.org/10.1080/10705511.2014.915181

Batchelor, S., Keeble, S., & Gilmore, C. (2015). Magnitude representations and counting skills in preschool children. Mathematical Thinking and Learning, 17(2-3), 116-135. https://doi.org/10.1080/10986065.2015.1016811

Bermejo, V. (1996). Cardinality development and counting. Developmental Psychology, 32(2), 263. https://doi.org/10.1037/0012-1649.32.2.263

Brueggemann, A., & Gable, S. (2018). Preschoolers' selective sustained attention and numeracy skills and knowledge. Journal of experimental child psychology, 171, 138-147. https://doi.org/10.1016/j.jecp.2018.02.001

Bruyer, R., & Brysbaert, M. (2011). Combining speed and accuracy in cognitive psychology: is the inverse efficiency score (IES) a better dependent variable than the mean reaction time (RT) and the percentage of errors (PE)? Psychologica Belgica, 51(1), 5-13. http://doi.org/10.5334/pb-51-1-5

Bull, R., Espy, K. A., & Wiebe, S. A. (2008). Short-term memory, working memory, and executive functioning in preschoolers: Longitudinal predictors of mathematical achievement at age 7 years. Developmental Neuropsychology, 33(3), 205 228. https://doi.org/10.1080/87565640801982312

Cahoon, Cassidy, Purpura & Simms. (in review). Developing a rigorous measure of the preschool home maths environment. Journal of Numerical Cognition.

Canobi, K. H. (2005). Children's profiles of addition and subtraction understanding. Journal of Experimental Child Psychology, 92(3) 220- 246 https://doi.org/10.1016/j.jecp.2005.06.001

Chu, F. W., vanMarle, K., & Geary, D. C. (2016). Predicting children's reading and mathematics achievement from early quantitative knowledge and domain-general cognitive abilities. *Frontiers in Psychology*, *7*, 775. https://doi.org/10.3389/fpsyg.2016.00775

Condry, K. F., & Spelke, E. S. (2008). The development of language and abstract concepts: The case of natural number. Journal of Experimental Psychology: General, 137(1), 22. https://doi.org/10.1037/0096-3445.137.1.22

De Smedt, B., Taylor, J., Archibald, L., & Ansari, D. (2010). How is phonological processing related to individual differences in children's arithmetic skills?. Developmental science, 13(3), 508-520. https://doi.org/10.1111/j.1467-7687.2009.00897.x

Dearing, E., Casey, B. M., Ganley, C. M., Tillinger, M., Laski, E., & Montecillo, C. (2012). Young girls' arithmetic and spatial skills: The distal and proximal roles of family socioeconomics and home learning experiences. Early Childhood Research Quarterly, 27(3):458–70. https://doi.org/10.1016/j.ecresq.2012.01.002

Dowker, A. (2008). Individual differences in numerical abilities in preschoolers. Developmental Science, 11(5), 650-654. https://doi.org/10.1111/j.1467-7687.2008.00713.x

Duncan, G.J., Dowsett, C.J., Claessens, A., Magnuson, K., Huston, A.C., Klebanov, P., Pagani, L.S., Feinstein, L., Engel, M., Brooks-Gunn, J. & Sexton, H. (2007). School readiness and later achievement. Developmental Psychology, 43(6), 1428. https://doi.org/10.1037/0012-1649.43.6.1428

Dunn, L. M., & Dunn, D. M. (2009). The British picture vocabulary scale. GL Assessment Limited.

Dunn, L. M., Dunn, D. M., Styles, B., & Sewell, J. (2009). The British Picture Vocabulary Scale III – 3rd Edition. London: GL Assessment

Dziak, J. J., Lanza, S. T., & Tan, X. (2014). Effect size, statistical power, and sample size requirements for the bootstrap likelihood ratio test in latent class analysis. Structural equation modeling: a multidisciplinary journal, 21(4), 534-552. https://doi.org/10.1080/10705511.2014.919819

Elliot, C., Smith, P., & McCulloch, K. (1996). British Ability Scales Second Edition (BAS II). Windsor, UK: NFER-Nelson Publishing Company Limited.

Fryer, L. K. (2017). (Latent) transitions to learning at university: A latent profile transition analysis of first-year Japanese students. Higher Education, 73(3), 519-537. https://doi.org/10.1007/s10734-016-0094-9

Garon-Carrier, G., Boivin, M., Lemelin, J. P., Kovas, Y., Parent, S., Séguin, J. R., ... & Dionne, G. (2018). Early developmental trajectories of number knowledge and math achievement from 4 to 10 years: Low-persistent profile and early-life predictors. *Journal of school psychology*, *68*, 84-98. https://doi.org/10.1016/j.jsp.2018.02.004

Geary, D. C., vanMarle, K., Chu, F. W., Rouder, J., Hoard, M. K., & Nugent, L. (2018). Early conceptual understanding of cardinality predicts superior school-entry number-system knowledge. *Psychological science*, *29*(2), 191-205. DOI: 10.1177/0956797617729817

Geiser, C. (2012). Data analysis with Mplus. Guilford press.

Gelman, R., & Gallistel, C. (1978). Young children's understanding of numbers. Cambridge, MA.

Gilmore, C. K., & Bryant, P. (2006). Individual differences in children's understanding of inversion and arithmetical skill. British Journal of Educational Psychology, 76(2), 309-331. https://doi.org/10.1348/000709905X39125

Göbel, S. M., Watson, S. E., Lervåg, A., & Hulme, C. (2014). Children’s arithmetic development: It is number knowledge, not the approximate number sense, that counts. Psychological Science, 25(3), 789-798. https://doi.org/10.1177/0956797613516471

Grammer, J. K., Coffman, J. L., Ornstein, P. A., & Morrison, F. J. (2013). Change over time: Conducting longitudinal studies of children's cognitive development. Journal of Cognition and Development, 14(4), 515-528. https://doi.org/10.1080/15248372.2013.833925

Guy, J., Rogers, M., & Cornish, K. (2013). Age-related changes in visual and auditory sustained attention in preschool-aged children. Child Neuropsychology, 19(6), 601-614. https://doi.org/10.1080/09297049.2012.710321

Hannant, P. (2018). Receptive language is associated with visual perception in typically developing children and sensorimotor skills in autism spectrum conditions. Human movement science, 58, 297-306. https://doi.org/10.1016/j.humov.2018.03.005

Hickendorff, M., Edelsbrunner, P. A., McMullen, J., Schneider, M., & Trezise, K. (2017). Informative tools for characterizing individual differences in learning: Latent class, latent profile, and latent transition analysis. Learning and Individual Differences. https://doi.org/10.1016/j.lindif.2017.11.001

Howse, R. B., Lange, G., Farran, D. C., & Boyles, C. D. (2003). Motivation and self-regulation as predictors of achievement in economically disadvantaged young children. The Journal of Experimental Education, 71(2), 151-174. https://doi.org/10.1080/00220970309602061

Jordan, N. C., Hanich, L. B., & Kaplan, D. (2003). Arithmetic fact mastery in young children: A longitudinal investigation. *Journal of experimental child psychology*, *85*(2), 103-119. http://dx.doi.org/10.1016/S0022-0965(03)00032-8.

Jordan, N. C., Kaplan, D., Locuniak, M. N., & Ramineni, C. (2007). Predicting first-grade math achievement from developmental number sense trajectories. Disabilities Research and Practice, 22, 36–46. https://doi.org/10.1111/j.1540-5826.2007.00229.x

Jordan, N. C., Kaplan, D., Nabors Oláh, L., & Locuniak, M. N. (2006). Number sense growth in kindergarten: A longitudinal investigation of children at risk for mathematics difficulties. Child Development, 77(1), 153-175. https://doi.org/10.1111/j.1467-8624.2006.00862.x

Kim, D. J., Ferrini-Mundy, J., & Sfard, A. (2012). How does language impact the learning of mathematics? Comparison of English and Korean speaking university students' discourses on infinity. International Journal of Educational Research, 51, 86-108. https://doi.org/10.1016/j.ijer.2012.01.004

Kleemans, T., Peeters, M., Segers, E., & Verhoeven, L. (2012). Child and home predictors of early numeracy skills in kindergarten. Early Childhood Research Quarterly, 27(3), 471-477. https://doi.org/10.1016/j.ecresq.2011.12.004

Landerl, K. (2013). Development of numerical processing in children with typical and dyscalculic arithmetic skills—a longitudinal study. Frontiers in Psychology, 4, 459. https://doi.org/10.3389/fpsyg.2013.00459

Lanza, S. T., & Cooper, B. R. (2016). Latent class analysis for developmental research. Child Development Perspectives, 10(1), 59-64. https://doi.org/10.1111/cdep.12163

Le Corre, M., & Carey, S. (2007). One, two, three, four, nothing more: An investigation of the conceptual sources of the verbal counting principles. Cognition, 105(2), 395-438. https://doi.org/10.1016/j.cognition.2006.10.005

LeFevre, J. A., Skwarchuk, S. L., Smith-Chant, B. L., Fast, L., Kamawar, D., & Bisanz, J. (2009). Home numeracy experiences and children's math performance in the early school years. Canadian Journal of Behavioural Science, 41(2), 55. https://doi.org/10.1037/a0014532

Li, Y., Zhang, M., Chen, Y., Deng, Z., Zhu, X., & Yan, S. (2018). Children's Non-symbolic and Symbolic Numerical Representations and Their Associations With Mathematical Ability. Frontiers in Psychology, 9, 1035. https://doi.org/10.3389/fpsyg.2018.01035

Lindblom-Ylänne, S., Parpala, A., & Postareff, L. (2015). Methodological challenges in measuring change in students' learning processes. World Education Research, 160-183. https://doi.org/10.1080/03075079.2018.1482267

Lira, C. J., Carver, M., Douglas, H., & LeFevre, J. A. (2017). The integration of symbolic and non-symbolic representations of exact quantity in preschool children. Cognition, 166, 382-397. https://doi.org/10.1016/j.cognition.2017.05.033

Lyons, I. M., & Ansari, D. (2015). Numerical order processing in children: From reversing the distance-effect to predicting arithmetic. *Mind, Brain, and Education*, *9*(4), 207-221. https://doi.org/10.1111/mbe.12094

Lyons, I. M., & Beilock, S. L. (2011). Numerical ordering ability mediates the relation between number-sense and arithmetic competence. Cognition, 121(2), 256-261. https://doi.org/10.1016/j.cognition.2011.07.009

Lyons, I. M., Price, G. R., Vaessen, A., Blomert, L., & Ansari, D. (2014). Numerical predictors of arithmetic success in grades 1–6. Developmental Science, 17(5), 714-726. https://doi.org/10.1111/desc.12152

Mahone, E. M., Pillion, J. P., & Hiemenz, J. R. (2001). Initial development of an auditory continuous performance test for preschoolers. Journal of Attention Disorders, 5(2), 93-106. https://doi.org/10.1177/108705470100500203

Manolitsis, G., Georgiou, G. K., & Tziraki, N. (2013). Examining the effects of home literacy and numeracy environment on early reading and math acquisition. Early Childhood Research Quarterly, 28(4), 692-703. https://doi.org/10.1016/j.ecresq.2013.05.004

Marsh, H. W., Lüdtke, O., Trautwein, U., & Morin, A. J. (2009). Classical latent profile analysis of academic self-concept dimensions: Synergy of person-and variable-centered approaches to theoretical models of self-concept. Structural Equation Modeling, 16(2), 191-225. https://doi.org/10.1080/10705510902751010

McClelland, M. M., Morrison, F. J., & Holmes, D. L. (2000). Children at risk for early academic problems: The role of learning-related social skills. Early Childhood Research Quarterly, 15(3), 307-329. https://doi.org/10.1016/S0885-2006(00)00069-7

McCormack, T., Simms, V., McGourty, J., & Beckers, T. (2013). Blocking in children's causal learning depends on working memory and reasoning abilities. Journal of Experimental Child Psychology, 115(3), 562-569. https://doi.org/10.1016/j.jecp.2012.11.016

Missall, K., Hojnoski, R. L., Caskie, G. I., & Repasky, P. (2015). Home numeracy environments of preschoolers: Examining relations among mathematical activities, parent mathematical beliefs, and early mathematical skills. Early Education and Development, 26(3), 356-376. https://doi.org/10.1080/10409289.2015.968243

Moll, K., Snowling, M. J., Göbel, S. M., & Hulme, C. (2015). Early language and executive skills predict variations in number and arithmetic skills in children at family-risk of dyslexia and typically developing controls. Learning and Instruction, 38, 53-62. https://doi.org/10.1016/j.learninstruc.2015.03.004

Mundy, E., & Gilmore, C. K. (2009). Children's mapping between symbolic and nonsymbolic representations of number. Journal of Experimental Child Psychology, 103(4), 490-502. https://doi.org/10.1016/j.jecp.2009.02.003

Nylund-Gibson, K., Grimm, R., Quirk, M., & Furlong, M. (2014). A latent transition mixture model using the three-step specification. Structural Equation Modeling: A Multidisciplinary Journal, 21(3), 439-454. https://doi.org/10.1080/10705511.2014.915375

Okamoto, Y., & Case, R. (1996). II. Exploring the microstructure of children's central conceptual structures in the domain of number. *Monographs of the Society for research in Child Development*, *61*(1-2), 27-58. DOI: 10.1111/j.1540-5834.1996.tb00536.x

Olson, S. L., Sameroff, A. J., Kerr, D. C., Lopez, N. L., & Wellman, H. M. (2005). Developmental foundations of externalizing problems in young children: The role of effortful control. Development and Psychopathology, 17(1), 25-45. https://doi.org/10.1017/S0954579405050029

Peng, P., Namkung, J., Barnes, M., & Sun, C. (2016). A meta-analysis of mathematics and working memory: Moderating effects of working memory domain, type of mathematics skill, and sample characteristics. Journal of Educational Psychology, 108(4), 455. https://doi.org/10.1037/edu0000079

Purpura, D. J., Hume, L. E., Sims, D. M., & Lonigan, C. J. (2011). Early literacy and early numeracy: The value of including early literacy skills in the prediction of numeracy development. *Journal of Experimental Child Psychology*, *110*(4), 647-658. https://doi.org/10.1016/j.jecp.2011.07.004

Rose, D., & Pevalin, D. (2010). Volume 3 The National Statistics Socio-economic Classification:(rebased on the SOC2010) User manual (pp. 1–70).

Sammons, P., Elliot, K., Sylva, K., Melhuish, E. C., Siraj-Blatchford, I., & Taggart, B. (2004). The impact of pre-school on young children's cognitive attainments at entry to reception. *British Educational Research Journal, 30*, 691–712. https://doi.org/10.1080/0141192042000234656

Sarnecka, B. W., & Carey, S. (2008). How counting represents number: What children must learn and when they learn it. *Cognition*, *108*(3), 662-674. https://doi.org/10.1016/j.cognition.2008.05.007

Sarnecka, B. W., & Wright, C. E. (2013). The idea of an exact number: Children's understanding of cardinality and equinumerosity. *Cognitive Science*, *37*(8), 1493-1506. https://doi.org/10.1111/cogs.12043

Schneider, M., & Hardy, I. (2013). Profiles of inconsistent knowledge in children's pathways of conceptual change. *Developmental Psychology*, *49*(9), 1639. https://doi.org/10.1037/a0030976

Sharp, C. (2002, November). School starting age: European policy and recent research. In *LGA Seminar: When Should Our Children Start School*.

Skwarchuk, S. L., Sowinski, C., & LeFevre, J. A. (2014). Formal and informal home learning activities in relation to children's early numeracy and literacy skills: The development of a home numeracy model. *Journal of Experimental Child Psychology*, *121*, 63-84. https://doi.org/10.1016/j.jecp.2013.11.006

Slusser, E. B., & Sarnecka, B. W. (2011). Find the picture of eight turtles: A link between children's counting and their knowledge of number word semantics. *Journal of Experimental Child Psychology*, *110*(1), 38-51. https://doi.org/10.1016/j.jecp.2011.03.006

Townsend, J. T., & Ashby, F. G. (1978). Methods of modeling capacity in simple processing systems. *Cognitive Theory*, *3*, 200-239.

Vukovic, R. K., & Lesaux, N. K. (2013). The language of mathematics: Investigating the ways language counts for children's mathematical development. *Journal of Experimental Child Psychology*, *115*(2), 227-244. http://dx.doi.org/10.1016/j.jecp.2013.02.002

Wagner, J. B., & Johnson, S. C. (2011). An association between understanding cardinality and analog magnitude representations in preschoolers. *Cognition*, *119*(1), 10-22. https://doi.org/10.1016/j.cognition.2010.11.014

Wright, R. J., Martland, J., & Stafford, A. K. (2006). Early numeracy: Assessment for teaching and intervention. Sage.

Wurpts, I. C., & Geiser, C. (2014). Is adding more indicators to a latent class analysis beneficial or detrimental? Results of a Monte-Carlo study. *Frontiers in Psychology*, *5*, 920. https://doi.org/10.3389/fpsyg.2014.00920

Wynn, K. (1990). Children's understanding of counting. *Cognition, 36*(2), 155-193. https://doi.org/10.1016/0010-0277(90)90003-3

Xenidou-Dervou, I., Van Luit, J. E., Kroesbergen, E. H., Friso-van den Bos, I., Jonkman, L. M., van der Schoot, M., & Van Lieshout, E. C. (2018). Cognitive predictors of children's development in mathematics achievement: A latent growth modeling approach. *Developmental Science*, *21*(6), e12671. https://doi.org/10.1111/desc.12671

Table 1. *Socioeconomic classification and educational attainment breakdown*

| | Categories | Participants *n* (%) (n = 128) |
|---|---|---|
| Socioeconomic classification | Higher managerial, administrative and professional occupations | 50 (39) |
| | Intermediate occupations | 27 (21) |
| | Routine and manual occupations | 30 (23.4) |
| Highest Educational Qualification | Middle secondary school level qualifications (e.g. GCSEs / O level / Irish Junior Certificate) | 25 (19.5) |
| | Higher secondary school level qualifications (e.g. A-levels / BTEC / Irish Leaving Certificate) | 18 (14.1) |
| | Higher education qualifications (e.g. degree, masters or PhD) | 58 (45.4) |
| | No qualifications | 9 (7.0) |

Table 2. *Descriptive statistics of each measure across time*

| Measure | Scoring type | | Mean | Std. Deviation | Sig. change | No. of children at floor | No. of children at ceiling |
|---|---|---|---|---|---|---|---|
| Digit recognition | Proportion score | Time 1 | .41 | .31 | <.01 | 19 | 8 |
| | | Time 2 | .47 | .31 | | 11 | 13 |
| | | Time 3 | .60 | .31 | | 1 | 24 |
| Cross-notation comparison | Proportion score | Time 1 | .64 | .21 | <.01 | 1 | 7 |
| | | Time 2 | .74 | .21 | | 1 | 23 |
| | | Time 3 | .83 | .17 | | 0 | 35 |
| Give-N | Proportion score | Time 1 | .49 | .35 | <.01 | 20 | 11 |
| | | Time 2 | .56 | .35 | | 24 | 14 |
| | | Time 3 | .69 | .26 | | 4 | 12 |
| Numeral ordering | Proportion score | Time 1 | .34 | .30 | <.01 | 26 | 6 |
| | | Time 2 | .40 | .35 | | 27 | 15 |
| | | Time 3 | .60 | .35 | | 9 | 33 |
| British Picture Vocabulary Scale (BPVS) | Raw score | Time 1 | 57.29 | 16.92 | <.01 | 0 | 0 |
| | | Time 2 | - | - | - | - | - |
| | | Time 3 | 69.58 | 14.22 | | 0 | 0 |
| Auditory Continuous Performance Test – Preschool | Inverse efficiency score | Time 1 | 3.29 | 5.79 | <.01 | 0 | 30 |
| | | Time 2 | 2.31 | 2.90 | | 0 | 35 |
| | | Time 3 | 2.27 | 6.51 | | 0 | 38 |
| Verbal animal recall | Accuracy score | Time 1 | 4.04 | 2.91 | <.01 | 16 | 0 |
| | | Time 2 | 5.28 | 3.49 | | 4 | 0 |
| | | Time 3 | 6.21 | 3.41 | | 6 | 0 |

Note: * N = 128 ** N = 118

Table 3. *Latent transition analysis fit statistics (models 2-5 represent solutions with 2-5 profiles)*

| Fit criterion | Models | | | |
|---|---|---|---|---|
| | 2 | 3 | 4 | 5 |
| Akaike information criterion (AIC) | -297.481 | -570.880 | -660.820 | -728.865 |
| Bayesian information criterion (BIC) | -180.548 | -394.055 | -412.694 | -398.030 |
| Sample size - adjusted BIC (aBIC) | -310.212 | -590.133 | -687.836 | -764.886 |
| Entropy | 0.962 | 0.951 | 0.939 | 0.957 |

Table 4. *Means of child measures according to pathways*

| Variable | Pathway 1 | Pathway 2 | Pathway 3 | Pathway 4 | Pathway 5 |
|---|---|---|---|---|---|
| T1 Receptive vocabulary | 41.56 | 50.68 | 57.07 | 60.27 | 68.39 |
| T1 Working memory | 2.63 | 2.97 | 4.33 | 3.58 | 5.76 |

Note: * $p < .05$ ** $p < .01$ (two-tailed).

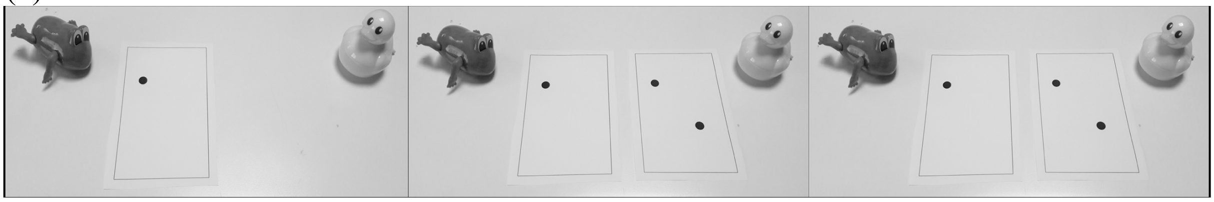
Table 5. *Adjusted associations between pathway membership and demographics and child measures predictors*

| | Pathway 1 – Consistently low number skills pathway | | Pathway 2 - Low to intermediate number skills shifting pathway (at school-entry) | | Pathway 3 – Consistently intermediate number skills pathway | | Pathway 4 – Intermediate to advanced number skills shifting pathway (at school-entry) | |
|---|---|---|---|---|---|---|---|---|
| | AOR | p Value | AOR | p Value | AOR | p Value | AOR | p Value |
| Age at T1 (mo.) | **0.608** | 0.019* | **0.807** | 0.012* | 0.812 | 0.093 | 0.939 | 0.495 |
| SES | | | | | | | | |
| High | Reference category | | | | | | | |
| Middle | 0.000 | 0.632 | 1.346 | 0.682 | 0.514 | 0.442 | 0.747 | 0.678 |
| Low | 3.223 | 0.347 | 0.678 | 0.631 | 0.131 | 0.103 | 0.321 | 0.264 |
| Higher Education Qualification | | | | | | | | |
| GCSE | Reference category | | | | | | | |
| A-level | 0.088 | 0.061 | **0.045** | 0.008** | **0.057** | 0.034* | **0.053** | 0.011* |
| Degree | **0.010** | 0.011* | **0.051** | 0.008** | **0.013** | < 0.001*** | **0.079** | 0.031* |
| Masters | 0.000 | 0.306 | **0.016** | < 0.001*** | **0.026** | 0.007** | **0.038** | 0.010* |
| PhD | 0.506 | 0.744 | **0.045** | 0.036* | 0.000 | 0.319 | 0.056 | 0.053 |
| No qualification | **0.022** | 0.010* | **0.022** | 0.007** | 0.056 | 0.078 | **0.040** | 0.025* |
| T1 Receptive vocabulary | **0.889** | < 0.001*** | **0.930** | 0.005** | 0.951 | 0.068 | 0.973 | 0.214 |
| T1 Working memory | 0.765 | 0.138 | **0.756** | 0.020* | 0.912 | 0.340 | **0.785** | 0.010** |

Reference pathway: Pathway 5 the consistently advanced number skills pathway. AOR, adjusted odds ratio. Adjusted associations from multivariate multinomial logistic regression. Note: * p < .05, ** p < .01, *** p <.001 (two-tailed). Bold = p < .05. N = 126.

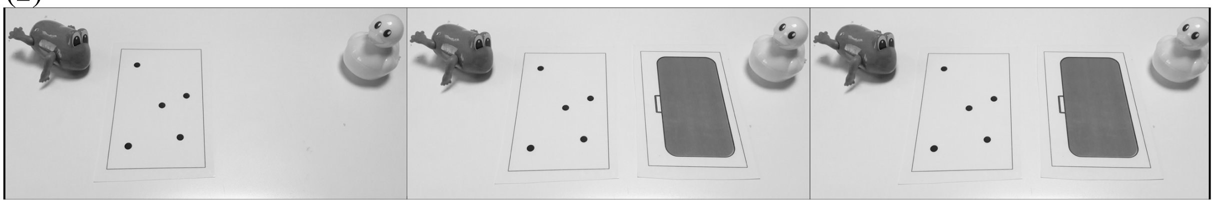
*Figure 1. Set-up of the cross-notation comparison task (1) is an example of the practice trial which was a non-symbolic (dot comparison) and (2) is an example of a cross-notation comparison trial.*

(1)



| | | |
|---|---|---|
| "The frog has this many balls." | "The duck has this many balls." | "Who has the most balls?". |

(2)



| | | |
|---|---|---|
| "The frog has this many balls." | "The duck has hidden their balls in a box. The duck has four balls." | "Who has the most balls?". |

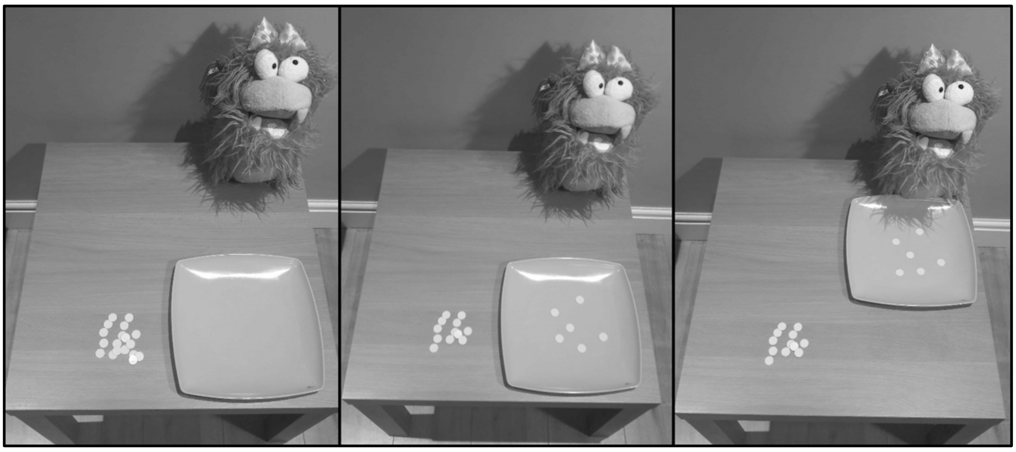
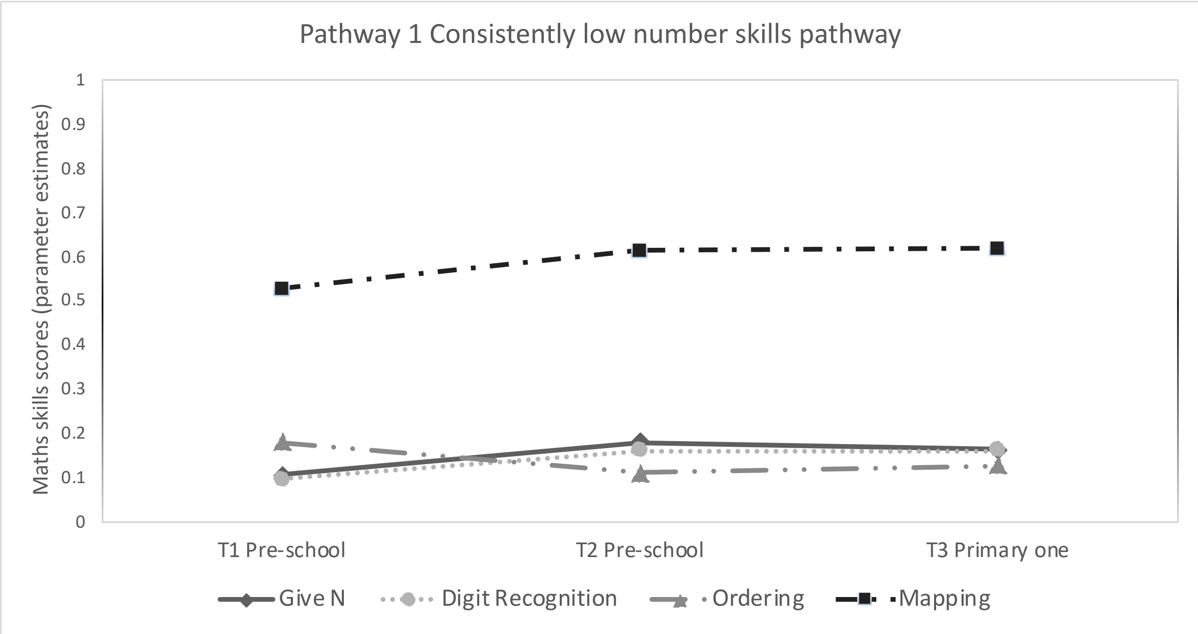
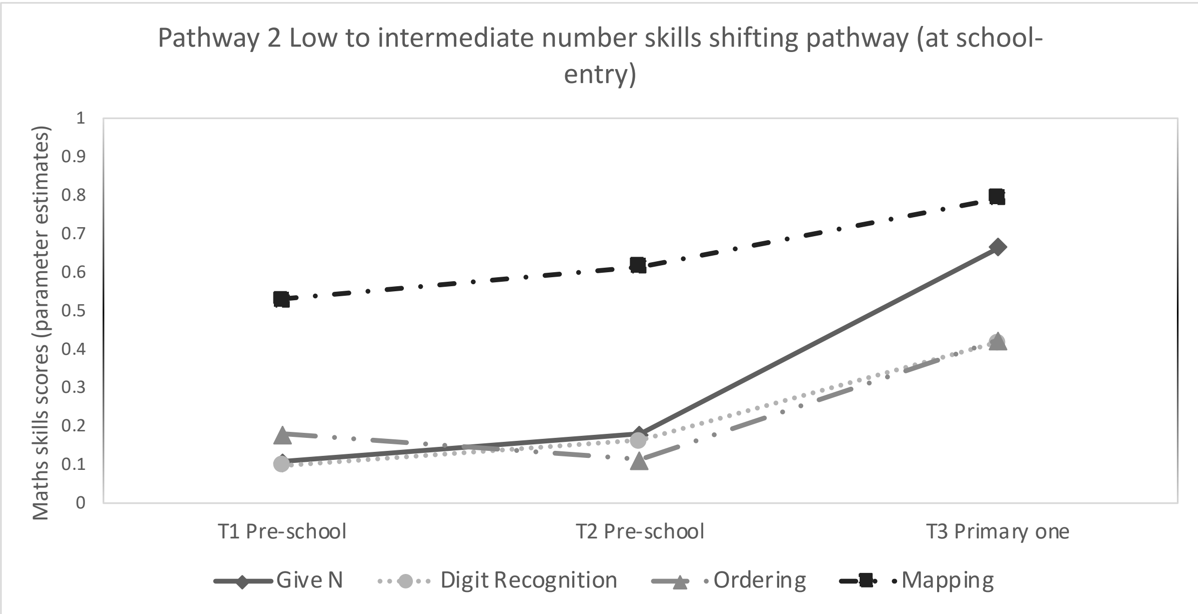
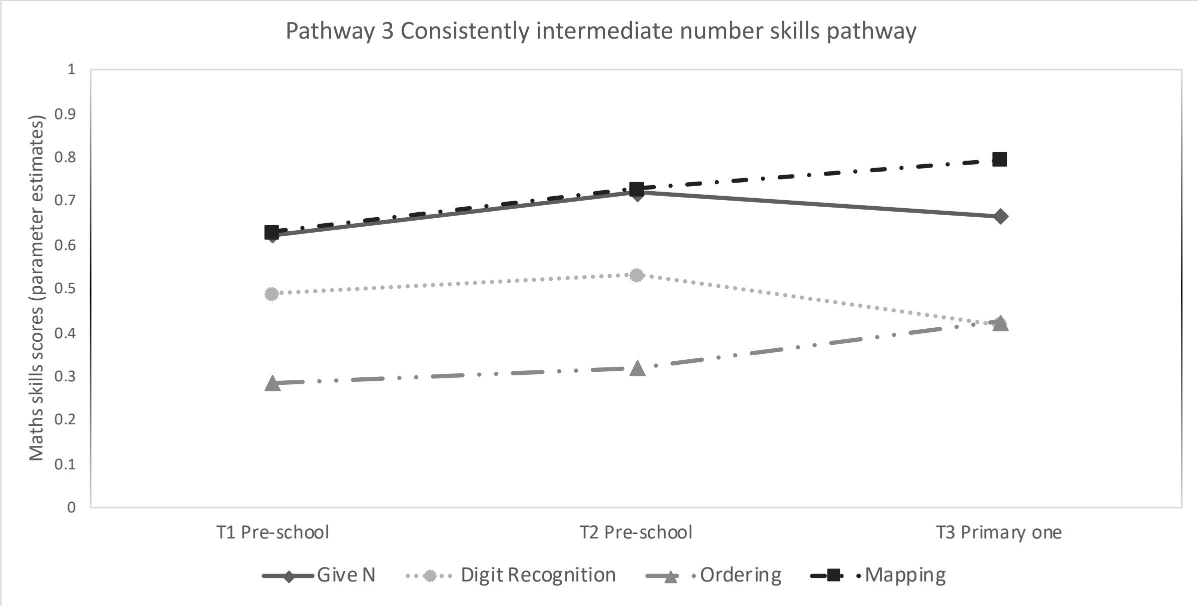


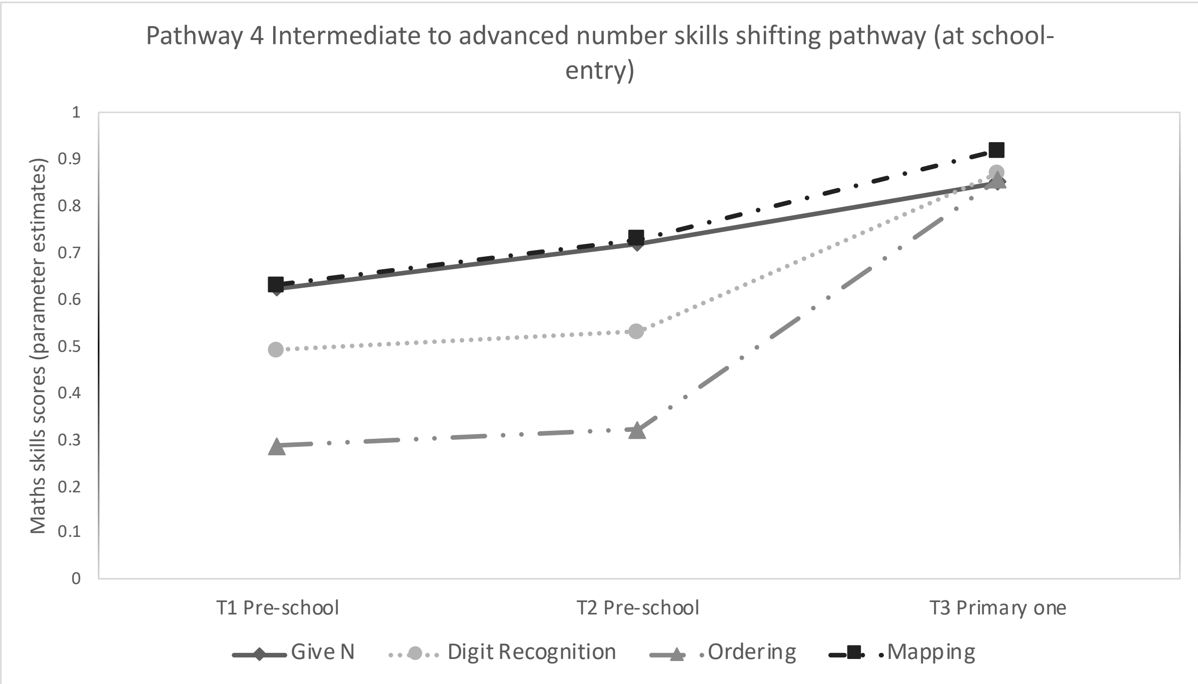
*Figure* 2. *Showing the experimental set-up of the Give-N task*
Step 1. Illustrates the set up at the start of the task. Step 2. Demonstrates the counters being placed on the plate by the child. Step 3. Shows how to researcher knows the child is finished placing the counters on the plate, with the plate slide across and sitting in front of the puppet.

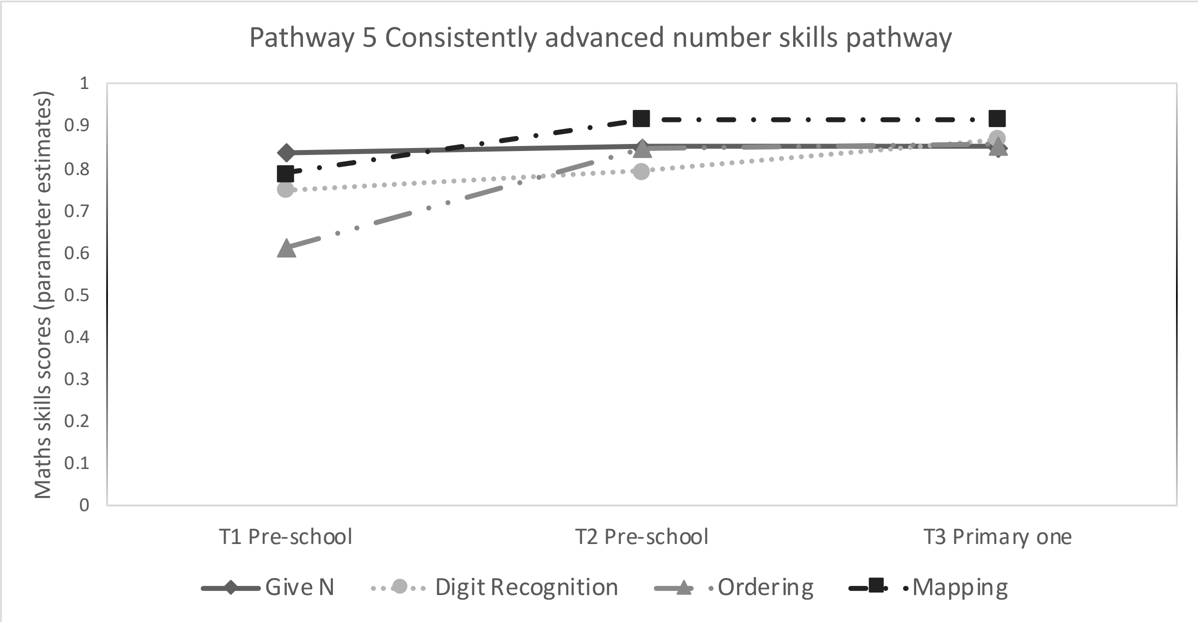
*Figure 3.* Pathway characteristics over time




Pathway 3 Consistently intermediate number skills pathway
Maths skills scores (parameter estimates)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
T1 Pre-school
T2 Pre-school
T3 Primary one
Give N
Digit Recognition
Ordering
Mapping


Pathway 4 Intermediate to advanced number skills shifting pathway (at school-entry)
Maths skills scores (parameter estimates)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
T1 Pre-school
T2 Pre-school
T3 Primary one
Give N
Digit Recognition
Ordering
Mapping




Note: Mapping task was forced choice with chance performance = 50%.

*Figure* 4. Diagram of the five transitional pathways taken by 98.4% of the population over the three time points. The numbers represented in the diagram are the actual number of participants transitioning and not percentages (n = 126)