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Abstract

Understanding of mathematical language is critical for numeracy skill development. However, prior research has focused exclusively on relations between mathematical language and numeracy skills, broadly measured. Thus, the aim of the current study was to explore more targeted relations between preschoolers' mathematical language and specific numeracy skills (e.g., cardinality, numeral comparison). Participants included 124 preschoolers, ages 3.52 to 6.03 years (M = 4.78 years, SD = 0.53). Children were assessed on a battery of early numeracy skills and mathematical language, as well as expressive vocabulary. Mixed-effects regression models were conducted, with school as a random effect and age, gender, and parental education as fixed effects covariates. Results indicated that mathematical language was significantly related to most numeracy skills, including verbal counting, one-to-one correspondence, numeral identification, cardinality, comparisons of sets and/or numerals, ordering numerals, and story problems. As hypothesized, mathematical language was not significantly related to either subitizing or formal addition, as these skills are independent of general language ability. Importantly, mathematical language was generally more proximal to each of these numeracy skills than was general language. These results provide direction for future research, in particular for the development of more precise measures to identify children at risk for mathematics difficulties as well as the incorporation of focused mathematical language instruction within early mathematics interventions.

Keywords: mathematical language, numeracy, mathematics, vocabulary, preschool

Highlights

- Past work has found math language is related to numeracy skills, broadly measured
- This study examined relations between math language and specific numeracy skills
- Math language is related to most numeracy skills
- Math language is not related to subitizing or formal addition skills
- Math language is more proximal to most numeracy skills than is general language

Relations Between Preschoolers' Mathematical Language Understanding and Specific Numeracy Skills

Children's early mathematical skills are important for later academic achievement (Duncan et al., 2007; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Watts, Duncan, Siegler, & Davis-Kean, 2014). It is evident from prior literature that understanding of mathematical language is critical for numeracy skill development (Purpura & Logan, 2015; Purpura & Reid, 2016; Toll & Van Luit, 2014a, 2014b). However, prior research has focused on relations between mathematical language and broad measures of numeracy skills even though measures of numeracy skills are comprised of more targeted skills (e.g., comparing quantities, cardinality). To date, no study has examined the relations between understanding of mathematical language and specific numeracy skills (e.g., numeral identification, connecting numerals to quantities). Just as cognitive skills such as working memory capacity and response inhibition may be differentially related to various early mathematics skills (Lan, Legare, Cameron Ponitz, Li, & Morrison, 2011; Purpura, Schmitt, & Ganley, 2017), children's understanding of mathematical language may be differentially related to specific early numeracy skills. It is important to understand the skill-specific relations that exist in order to develop more precise measures to identify children at risk for mathematics difficulties, as well as design interventions that take these relations into account in order to improve numeracy skills in young children. Thus, the aim of the current study is to assess targeted relations between mathematical language and specific numeracy skills, above and beyond general language.

Defining Early Numeracy Skills and Mathematical Language

Children's early numeracy skills are categorized in a few interconnected domains: numbering, relations, and arithmetic operations (National Research Council, 2009). Numbering skills include knowledge of the rules of the counting sequence and the ability to flexibly obtain quantity (Purpura & Lonigan, 2013). This includes skills such as subitizing (enumerating small sets of objects without counting), verbal counting (learning to recite the verbal number word sequence), structured counting (counting with one-to-one correspondence), and cardinality (understanding that the last word counted of the set indicates the total quantity; Clements & Sarama, 2007; Gelman & Gallistel, 1978; Jordan, Kaplan, Oláh, & Locuniak, 2006). Relations skills involve knowledge of how two or more items (sets/numbers) are related and understanding the association between numbers on the mental number line. Many relations tasks require knowledge of Arabic numerals (e.g., numeral identification, connecting numerals to quantity, numeral comparison, and ordering numerals), whereas others require distinguishing quantities of objects (e.g., set comparison; Clements & Sarama, 2007; Starkey, Klein, & Wakeley, 2004). Arithmetic operations skills include knowledge of composition and decomposition of numbers, as well as an early understanding of addition and subtraction (Clements & Sarama, 2007; Fuson, 1988) both through story problems (Hiebert, Carpenter, & Moser, 1982; Levine, Jordan, & Huttenlocher, 1992) and formal addition problems with Arabic numerals (Jordan et al., 2006).

Distinct from these early numeracy skills is children's understanding of mathematical language. Mathematical language is children's understanding of key words in mathematics, including quantitative words such as "more" and "less" (Barner, Chow, & Yang, 2009) and spatial words such as "before" and "after" (Pruden, Levine, & Huttenlocher, 2011). Understanding of these specific mathematics terms is thought to facilitate understanding of more complex numerical concepts (Barner et al., 2009). Importantly, the understanding of these content-specific words such as "most" and "fewest" is separate from conceptual knowledge of exact quantities and numeracy skills (Lansdell, 1999). Thus, mathematical language skill is a separate construct considering knowledge of key words/concepts used in early mathematics, whereas early numeracy skills are indicative of children's ability to work with exact numbers (e.g., count, compare, add).

Relation Between Language and Numeracy Skills

In the early years, there is a robust relation between language and numeracy development (Kleemans, Segers, & Verhoeven, 2011; Purpura, Hume, Sims, & Lonigan, 2011). In particular, there is a strong correlation between general vocabulary and number-word knowledge in children 2 ½ to 5 years of age (Negen & Sarnecka, 2012). Language skills are believed to support numerical development, as children use language to refine their understanding of quantitative concepts (Spelke, 2003). For example, children in cultures with a language that has plural markers (e.g., English) acquire the concept of "one" more quickly than children in cultures with a language without plural markers (Sarnecka, Kamenskaya, Yamana, Ogura, & Yudovina, 2007). A recent theory of the development of natural number understanding posits that understanding of higher-order numerical concepts depend on language, because children learn number words and quantified noun phrases that propel understanding of number (Spelke, 2017).

Of the various early language and literacy skills, oral language skills are most related to numeracy skills (Purpura et al., 2011; Vukovic & Lesaux, 2013a, 2013b), and this is true for the vast majority of skills (LeFevre et al., 2010; Purpura & Ganley, 2014), with the exception of subitizing (Purpura & Ganley, 2014) and formal calculation (Purpura et al., 2011). Importantly, mathematical language skill is more proximal to children's numeracy skills than is general language (Purpura & Reid, 2016; Toll & Van Luit, 2014a, b). In fact, Toll and Van Luit (2014b) found that mathematical language significantly mediates the relation between general language and early numeracy skills. However, no prior study has examined relations between mathematical language and specific numeracy skills (rather than numeracy skills broadly), above and beyond general language. Thus, this study aimed to examine these targeted relations.

Relation Between Mathematical Language and Numeracy Skills

Both parent and teacher usage of mathematical language is predictive of growth in children's numeracy skills (Gunderson & Levine, 2011; Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006; Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010; Ramani, Rowe, Eason, & Leech, 2015). Prior work also demonstrates links between children's understanding of mathematical language and numeracy skills (Purpura & Logan, 2015; Purpura & Reid, 2016; Toll & Van Luit, 2014a, 2014b). In particular, within the preschool year, low performance on a mathematical language assessment in the fall is the most consistent classifier of low numeracy skills in the spring (Purpura, Day, Napoli, & Hart, 2017). Prior research has also determined that understanding of mathematical language is the greatest predictor of kindergarteners' growth in mathematics achievement after an intervention (Devlin, Hornburg, & McNeil, 2017). Taken together, prior work points to a relation between mathematical language and numeracy skills, but dependent measures have typically been one broad assessment of numeracy skills (e.g., Early Numeracy Test-Revised [Van Luit & Van de Rijt, 2009]; Preschool Early Numeracy Skills Test-Brief Version [PENS-B; Purpura, Reid, Eiland, & Baroody, 2015]). Although those assessments are valid in determining overall proficiency, one broad measure does not adequately allow for differentiation of relations to each dimension of numerical cognition. As noted, numeracy skills are comprised of a range of components that may be more or less related to general language, as well as content-specific mathematical language.

Differential Relations Between Mathematical Language and Specific Numeracy Skills

There are likely three distinct groups of numeracy subskills that should be differentially related to mathematical language. First, some numeracy skills are more basic skills that are procedural in nature and would not require the use of mathematical language aside from learning the names of numbers (e.g., verbal counting, one-to-one-correspondence, numeral identification); as such, these skills would likely not be related to mathematical language above and beyond general language. Second, there are several numeracy skills that build upon both prior knowledge of the basic procedural skills and understanding of mathematical language to achieve knowledge of the higher-order concept. These complex skills are likely related to mathematical language above and beyond general language (e.g., cardinality, connecting numerals to quantities, set comparison, numeral comparison, number order, and story problems). Finally, there are some numeracy skills that are independent of general language ability, and thus are likely to be unrelated to mathematical language (e.g., subitizing, formal addition). Each of these groups of numeracy skills are discussed below.

Numeracy skills related to general language only. General language is central to the development of early mathematics skills (LeFevre et al., 2010; Purpura et al., 2011). However, some early numeracy skills are hypothesized to only be related to general language ability, rather than specifically related to mathematical language understanding.

Verbal counting. Verbal counting, for instance, in which children are asked to count aloud as high as possible (one, two, three, and so on), can be acquired through rote memorization and does not require understanding of the deeper structure (e.g., knowing that three is one more than two). The ordinal relations among number words in the count list are not made explicit when the words are simply recited in order (Spelke, 2017). Thus, verbal counting was predicted to only be related to general language ability, rather than mathematical language specifically.

One-to-one correspondence. One-to-one correspondence (i.e., each number word is said when pointing to a different item; Gelman & Gallistel, 1978) is another basic principle children can learn without understanding quantitative words such as "more" or spatial words such as "last." In fact, some children can successfully count to produce every number in their count list but fail to acquire a conceptual understanding and cannot correctly judge words occurring later in the count list as referring to larger magnitudes (Davidson, Eng, & Barner, 2012).

Numeral identification. Another numeracy skill unlikely to be related to mathematical language in particular (beyond general language) is numeral identification, in which children are shown a flashcard with an Arabic numeral on it (e.g., 7) and are simply asked "What number is this?". Task performance is likely impacted by children's exposure to Arabic numerals and print knowledge (Purpura et al., 2011), rather than a deeper understanding of mathematical language.

Numeracy skills related to mathematical language. Children's general language skill is expected to be related to each of the following complex early numeracy skills, because for each skill children need to know names of numbers, connect the meaning of the number name with a numeral and/or a quantity, or make comparisons between numerals and/or quantities as they integrate knowledge of number words, quantities, and symbols (Purpura & Ganley, 2014). Beyond general language, in order for children to be able to manipulate numbers in complex ways, a foundation of mathematical language is necessary.

Cardinality. First, mathematical language was predicted to have a positive relation to tasks assessing cardinality, above and beyond general language. Children who receive more mathematical language input from parents develop an understanding of cardinality at an accelerated rate (Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010). Understanding cardinality is more than knowing the count list (i.e., one, two, three); it is a deeper understanding of the quantities associated with those number words (Gelman & Gallistel, 1978; Wynn, 1990). Understanding cardinality involves understanding quantity as a property of a set of items (Chang, Sandhofer, Adelchanow, & Rottman, 2011). As an understanding of mathematical language increases, children may learn to map the characteristic of quantity to the set as a whole. In line with this progression, children who struggle to understand quantifiers such as “most” or “some” also struggle to grasp the complex concept of cardinality (Barner et al., 2009). However, it should be noted that the task used by Barner and colleagues (2009) to assess understanding of quantifiers (e.g., “Could you put most of the bananas into the red circle?”, given eight plastic bananas) was in a mathematical context. Thus, it is not yet established how a broader range of mathematical language (including quantitative and spatial language), specifically in a nonnumerical context, would be related to children’s understanding of cardinality.

Connecting numerals to quantities. Mathematical language was also predicted to have a significant positive relation to performance matching numerals to quantities (e.g., sets of objects). Children’s understanding of cardinality relates to their development of numerosity-based representations (Rousselle, Palmers, & Noël, 2004) and their ability to represent written numerals (Zhou & Wang, 2004). It is likely that this skill of connecting numerals to quantities (rather than simply connecting the name of the number to each numeral) is reflective of children’s understanding of cardinality. Because this skill is tapping into children’s understanding of cardinality, we hypothesized that children’s performance on the connecting numerals to quantities task would be related to children’s understanding of mathematical language above and beyond general language.

Set comparison and numeral comparison. Understanding of mathematical language is also likely related to set comparison, in which children reason about four sets of dots and determine which set has the most or the fewest number of dots, and numeral comparison, in which children identify which numbers (Arabic numerals or verbally spoken numbers) are the most or fewest. Clements and colleagues (2013) state within the learning trajectory for numerical relations that children need to learn vocabulary for “more” and “fewer” before they are able to accurately make verbal-based magnitude comparisons (Clements, Baroody, & Sarama, 2013). Because these tasks assess “most” and “fewest,” mathematical language was predicted to be related to both set comparison and numeral comparison above and beyond general language.

Number order. Performance on tasks in which children identify a missing number in a sequence (e.g., 3 4 5 __ 7 8, “What number comes after 5?”) is also likely related to the understanding of mathematical language, particularly spatial language. Although quantitative language (e.g., “fewest”) is not inherently needed to be successful on a number order task, spatial language understanding (e.g., “before”) should be related to performance, because that knowledge is essential to interpreting what is being asked, combined with knowledge of the verbal counting sequence. Thus, we predicted that mathematical language understanding would be related to children’s performance on a number order task above and beyond general language.

Story problems. Finally, understanding of the meaning of mathematical terms such as “more” and “take away” will help children succeed on story problems with simple scenarios of informal addition and subtraction. General language is a significant predictor of performance on word problems, as children need to process linguistic information to build models to solve the problem (Fuchs et al., 2008, 2010). In order to solve story problems correctly, children must first comprehend the problem before selecting an operation to perform and executing that calculation (Lewis & Mayer, 1987). Therefore, we predicted that mathematical language understanding would be related to performance on story problems above and beyond general language.

Numeracy skills unrelated to both general language and mathematical language.

Because strong general language ability is a prerequisite for the development of content-specific language such as mathematical language (Kleemans et al., 2011; Lansdell, 1999), it is likely that numeracy skills that are unrelated to general language are also unrelated to mathematical language.

Subitizing. Prior research has shown that subitizing, or rapid enumeration of small sets without counting, is unrelated to general language (Purpura & Ganley, 2014), in line with work demonstrating that subitizing is part of a system independent of language (Gelman & Butterworth, 2005). Children with specific language impairment perform no differently than typically developing children when quantifying two to six dots on a subitizing task (tapping into preverbal skills [Gallistel & Gelman, 1992]), but deficits emerge on complex sets (six to nine) requiring recall of number names (Donlan, 1993). Accordingly, subitizing was hypothesized to be unrelated to both general language ability and understanding of mathematical language.

Formal addition. Previous research has also demonstrated that language skills do not predict unique variance in children's performance on procedural calculation assessments (Fuchs et al., 2008, 2010). In fact, some children are able to carry out addition procedures given a written expression but cannot succeed on story problems which require comprehension of the language used (Mayer & Hegarty, 1996), which suggests that calculation skills are distinct from language skills. Given that past work has found that preschoolers' vocabulary knowledge does not account for unique variance in calculation skill (Purpura et al., 2011), children's performance on formal addition tasks (e.g., "What is 1 + 2?") was hypothesized to be unrelated to both general language ability and understanding of mathematical language.

The Present Study

The present study assesses the relations between preschoolers' understanding of mathematical language and specific numeracy skills. This study addresses a key gap in the literature, as only broad relations have been assessed previously, and it is unclear whether there are targeted relations between mathematical language and specific aspects of numeracy. In terms of skills with proposed relations to general language, we hypothesized two distinct categories of numeracy skills: those that are procedural skills not related to mathematical language specifically (e.g., verbal counting, one-to-one correspondence, and numeral identification), and those that are more complex skills that will be significantly related to understanding of mathematical language above and beyond general language (e.g., cardinality, connecting numerals to quantities, set comparison, numeral comparison, number order, and story problems). Finally, we hypothesized that general language would not be related to subitizing skills or formal addition skills, and accordingly, mathematical language would not be related to those skills either.

Method

Participants

Children with parental consent included 132 preschoolers recruited from 18 public and private preschools serving families with a range of socioeconomic backgrounds. Children were recruited from preschools in urban, suburban, and rural areas of a community in the Midwestern United States. Of those children, one did not assent to participate, one had speech delays that prevented assessment, and six left school before testing was completed, so the final sample consisted of 124 children. Children (46.0% female) ranged in age from 3.52 to 6.03 years old (M = 4.78 years, SD = 0.53). The race/ethnicity of the sample was approximately representative of the demographics of the area (80.5% Caucasian, 5.1% Asian, 5.1% Hispanic, 2.5% African American, and 6.8% multiracial or other race/ethnicity). All children were English speaking and had no known developmental disorders. Regarding parental education, 22.6% of children had parents with a high school education or less, 19.4% of children had parents with some college experience, 30.6% of children had at least one parent with a college degree, and 27.4% of children had at least one parent with a postgraduate degree.

Measures

Early numeracy. Each aspect of early numeracy mentioned above was assessed with a separate task described below (note that cardinality was assessed with two tasks). Measures were developed by Purpura and Lonigan (2015) and show evidence of reliability and validity. Children received one point for each correct response on all tasks. Internal consistency values reported below reflect scores from the present study.

Verbal counting. Children were asked to count as high as possible, starting from the number one. When a child made a mistake, or correctly counted to 100 without making a mistake, the task was stopped. Self-corrections were not scored as incorrect. The highest number counted to was converted to a score based on a 7-point scale. Children were awarded one point each for correctly counting to 5, 10, 15, 20, 25, 40, and 100.

One-to-one correspondence. Children were presented with a set of 3, 6, 11, 14, or 16 dots on a page and asked to count the set ($\alpha = .77$). Dots were arranged in one row for numbers 3, 6, and 11, and in two equal rows for numbers 14 and 16.

Numeral identification. Children were presented with flashcards of nine numerals (1, 2, 3, 7, 8, 10, 12, 14, and 15) ($\alpha = .87$) one at a time and asked, “What number is this?” Flashcards were presented in a random order.

Cardinality (how many). This skill was assessed in the context of the one-to-one correspondence task. At the completion of the counting 3, 6, and 11 one-to-one counting items, children were asked to indicate how many dots there were in all ($\alpha = .69$).

Cardinality (give-n). There were six items ($\alpha = .80$) for this task. Children were presented with a specific quantity of objects (e.g., 15) and asked to count out ("give me *n*") a smaller set of objects (e.g., 4) from the larger set. Set sizes to be counted out were 3, 4, 6, 8, 11, and 16.

Connecting numerals to quantities. This task had five items ($\alpha = .72$). For the first three items, children were presented with a numeral at the top of the page (e.g., 3) and five sets of dots below (e.g., |●●●●|●|●●●|●●|●●●●●|). They were instructed to identify which of the sets meant the same thing as the numeral at the top of the page. On the last two items, children were presented with a set of dots at the top of the page (e.g., ●●●●) and five numerals at the bottom (e.g., 4-2-3-1-5). They were instructed to identify which of the numerals meant the same thing as the set of dots at the top of the page.

Set comparison. For each of the six items ($\alpha = .72$), children were presented with four sets of dots on a page representing different quantities (e.g., |●●●|●●|●●●●●|●|). They were then asked which set had the most dots (three items) or fewest dots (three items).

Numeral comparison. Children were asked to identify which of four numerals was the biggest or smallest. There were six items ($\alpha = .71$); three were presented visually with Arabic numerals and three were presented verbally.

Number order. This task had six items ($\alpha = .84$). Children were shown a sequence of numbers with one number missing in between two numbers (e.g., 1 2 3 _ 5 6). They were asked what number comes before or after another number (e.g., "What number comes before 5?"). The items included identifying the numbers before 2, 5, and 15 and the numbers after 2, 9, and 15.

Story problems. This task had seven items (α = .74). Children were presented verbally with story problems that did not contain distractors (e.g., irrelevant information). These story problems were simple addition problems (three items) or subtraction problems (four items) that were relevant to children (e.g., "If Laura has two stickers and her mom gives her two more, how many stickers does she now have?").

Subitizing. Children were briefly presented (2 s) with a set of pictures (set sizes from 1 to 7 presented in a linear fashion, e.g., ●●●●) and instructed to say how many dots or pictures were presented. There were seven items (α = .52) on this task.

Formal addition. This task had five items (α = .78). Children were presented with a formal addition problem (e.g., 1 + 1 = …) and asked, "How much is …?"

Mathematical language. Understanding of mathematical language was assessed using Purpura and Logan's (2015) measure of mathematical content language. This measure includes 16 items assessing both quantitative (e.g., "more", "less") and spatial (e.g., "near", "far") language. In the development of this measure, items were selected from a larger battery of items using an item response theory framework (see Purpura & Logan, 2015). Selected items included a range of difficulty parameters and strong discrimination parameters. All items were designed to be completed in a non-numeracy context, without the need for exact quantitative skills (e.g., counting, adding). For example, understanding of "more" was assessed using gross comparisons of dots (e.g., 10 vs. 2). The measure of mathematical language included 6 quantitative items (take away, a little bit, more, less, most, and fewest) and 10 spatial items (nearest, under, first, far, below, front, middle, end, last, and before). Each item was read aloud to the children. The "take away" item was an expressive item that included manipulatives, whereas all other items were receptive items that used a testing manual with pictures. For the expressive item, children responded by moving blocks. For the receptive items, children responded either verbally or by pointing to one of multiple options. Children received one point for a correct response on each item. The mathematical language assessment had an internal consistency of .78 in our sample.

General language ability. Children's general language ability was assessed using the Expressive Vocabulary subtest from the *Clinical Evaluation of Language Fundamentals Preschool—Second Edition* (CELF:P2; Wiig, Secord, & Semel, 2004). Given the high correlations between this subtest and the CELF:P2 core language composite score (e.g., $r = .86$; Wiig et al., 2004) as well as the seemingly unidimensional structure of oral language skills in young children (Foorman, Herrera, Petscher, Mitchell, & Truckenmiller, 2015), the Expressive Vocabulary subtest was used as a proxy for general language ability. In this measure, children were presented with pictures of people, objects, and actions and were asked to name the picture (referential naming). The CELF:P2 has good reliability (α = .77-.84 for 3- to 5-year-old children; Wiig et al., 2004). Children received one point for each correct response. Basal and ceiling rules were administered per the instruction manual.

Covariates.

IQ. Children's IQ was assessed using the Slosson Intelligence Test Revised – Third Edition (SIT-R3; Slosson, Nicholson, & Hibpshman, 2005). This verbal measure assesses six cognitive domains: vocabulary, general information, similarities and differences, comprehension, quantitative, and auditory memory. The SIT-R3 has excellent reliability (α = .89-.90 for 4- to 5-year-old children; Slosson et al., 2005). Children received one point for each correct response. Basal and ceiling rules were administered per the instruction manual.

Demographic variables. Each model included school as a random-effect covariate to control for school-level effects. This was done because assessments occurred within an individual school at similar times, and schools varied in terms of curriculum used, so we added this effect to the models to account for school-level variance. Additionally, each model included three fixed-effects covariates of demographic variables. These included participant age, gender (male = 0, female = 1), and highest parental education (on a 9-point scale ranging from eighth grade or less to doctoral degree).

Procedure

Assessment procedure. Assessments took place throughout the preschool year, split into several 20- to 30-min sessions. Testing was conducted in shorter sessions as needed. All children were given the same specific assessments (e.g., expressive vocabulary) at approximately the same time of year. Assessments took place in the local preschools at times and locations designated by the schools' directors or teachers. Individuals who either had completed or were working toward completion of a bachelor's degree in a social science field conducted all assessments. These individuals completed two 2- to 3-hr training sessions, followed by individual practice and a "testing out" session to ensure that they were fluent with the assessment procedures and general principles of working with preschoolers.

Analytic procedure. To evaluate the relations of mathematical language with specific early numeracy skills, a series of separate mixed-effects regression analyses (Raudenbush & Bryk, 2002) was conducted. Covariates (school as a random effect; age, gender, and highest parental education as fixed effects) were included in all analyses. General language ability (as measured by expressive vocabulary) was also included as a fixed-effect predictor in each analysis. In our original set of models, IQ was included as a fixed-effect covariate as well; however, because our measure of IQ (a verbal measure) was highly correlated with both expressive vocabulary (r = .75) and mathematical language (r = .59), potential issues of multicollinearity emerged. Specifically, 10 of the 12 models had a negative coefficient for expressive vocabulary. A formal test of multicollinearity did not indicate that a large amount was present (*VIF* = 2.50 for IQ, *VIF* = 2.71 for expressive vocabulary, *VIF* = 1.98 for mathematical language). However, due to the negative coefficients observed for expressive vocabulary and our primary interest in examining both expressive vocabulary and mathematical language as predictors, IQ was removed from final models. However, note that we ran each model with and without IQ and when conclusions differed we note those below.

Because 8.9% of participants had missing data on at least one covariate or predictor variable (specifically, 1.5% of all data were missing), multiple imputation was conducted prior to analyses. Little's MCAR test was not significant, $\chi^2(97, N = 124) = 91.20$, $p = .65$, indicating that no identifiable pattern existed in the missing data. Imputation was conducted using the expectation maximization technique in the SPSS Missing Value Analysis program. Every analytic variable was included in the imputation model. Five data sets were imputed and aggregated for analyses. Benjamini-Hochberg corrections were applied within each regression analysis to adjust for multiple comparisons and reduce Type I error (Benjamini & Hochberg, 1995).

Results

Preliminary Analyses

Descriptive statistics for all measures are presented in Table 1 and zero-order correlations are presented in Table 2. With the exception of correlations between age and some numeracy skills measures, as well as parent education and some covariates and numeracy measures, all variables were significantly correlated.

Table 1

Descriptive Statistics

	Range	*M*	*SD*	Skew	Kurtosis
Age (in years)	3.52-6.03	4.78	0.53	-0.17	-0.22
Parent education	2-9	6.15	1.89	-0.16	-0.89
IQ (raw score)	5-61	29.32	11.44	0.06	0.001
Expressive vocabulary	2-38	21.35	8.05	-0.33	-0.37
Mathematical language	1-16	12.32	3.06	-1.19	1.00
Numeracy skills					
Verbal counting	0-7	3.02	1.86	0.15	-0.92
One-to-one correspondence	0-5	3.45	1.56	-0.72	-0.63
Numeral identification	0-9	5.67	2.68	-0.30	-1.04
Cardinality (how many)	0-3	2.22	0.97	-1.00	-0.12
Cardinality (give-*n*)	0-6	3.37	1.88	-0.31	-0.89
Connecting numerals to quantities	0-5	3.34	1.52	-0.56	-0.91
Set comparison	0-6	4.33	1.61	-0.65	-0.51
Numeral comparison	0-6	2.62	1.80	0.29	-0.93
Number order	0-6	3.15	2.05	-0.27	-1.19
Story problems	0-7	3.29	2.15	0.14	-1.10
Subitizing	0-7	3.94	1.36	-0.38	0.43
Formal addition	0-5	1.46	1.62	0.91	-0.38

Note. $N = 124$. Ranges presented for numeracy skills are both the observed and possible ranges. Ranges presented for other variables are the observed ranges

Table 2

Zero-Order Correlations

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.	17.
1. Age (in years)	_																
2. Parent education[a]	-.14	_															
3. IQ	.31	.17	_														
4. Expressive vocabulary	.29	.13	.75	_													
5. Mathematical language	.18	.36	.59	.64	_												
6. Verbal counting	.15	.30	.50	.40	.49	_											
7. One-to-one correspondence	.25	.34	.41	.35	.51	.57	_										
8. Numeral identification	.19	.29	.43	.34	.48	.55	.61	_									
9. Cardinality (how many)	.10	.26	.41	.38	.56	.43	.56	.49	_								
10. Cardinality (give-*n*)	.20	.35	.48	.44	.59	.68	.67	.70	.57	_							
11. Connecting numerals to quantities	.34	.11	.53	.42	.50	.44	.57	.58	.37	.60	_						
12. Set comparison	.30	.17	.58	.53	.58	.44	.39	.39	.47	.52	.53	_					
13. Numeral comparison	.28	.29	.48	.40	.52	.48	.48	.54	.44	.62	.44	.53	_				
14. Number order	.19	.30	.56	.42	.61	.63	.62	.76	.56	.71	.57	.45	.62	_			
15. Story problems	.10	.41	.55	.50	.56	.49	.43	.52	.45	.53	.40	.54	.59	.56	_		
16. Subitizing	.30	.11	.42	.34	.35	.39	.48	.45	.40	.38	.52	.41	.45	.46	.43	_	
17. Formal addition	.17	.17	.36	.26	.28	.34	.28	.39	.22	.38	.34	.28	.31	.40	.42	.26	_

Note. $N = 124$. All correlations except those underlined were significant at $p < .05$.
[a]1 = 8th grade or less, 2 = some high school, 3 = GED, 4 = high school diploma, 5 = some college, 6 = associate's degree, 7 = bachelor's degree, 8 = master's degree, 9 = doctoral/postgraduate degree. Correlations between parent education and each other variable are nonparametric (Spearman's rho) because parent education was an ordinal variable.

Primary Analyses

Results from each mixed-effects regression analysis are presented in Table 3. With the exception of verbal counting and formal addition, conditional intraclass correlation coefficients (ICCs) for all analyses were small (<.001-.17), suggesting that there was little systematic school-level variation for most numeracy skills. The ICCs for verbal counting and formal addition were .21 and .19, respectively. It is possible that some schools' curriculum may focus specifically on those particular formal skills more than other schools' curriculum.

Table 3

Mixed Effects Regression Analyses Predicting Specific Numeracy Skills

	R^2	*b*	*SE*	*F*	*p*
Verbal counting	.29				
Age		0.31	0.28	1.26	.263
Female		0.53	0.28	3.68	.058
Parent education		0.04	0.09	0.20	.654
Expressive vocabulary		0.03	0.02	1.96	.164
Mathematical language		0.16	0.06	6.27	.014
One-to-one correspondence	.32				
Age		0.61	0.23	6.89	.010
Female		-0.02	0.24	0.005	.945
Parent education		0.16	0.07	5.24	.024
Expressive vocabulary		0.008	0.02	0.19	.667
Mathematical language		0.17	0.05	10.18	.002

Numeral identification	.26				
Age		0.62	0.42	2.19	.141
Female		-0.24	0.44	0.30	.582
Parent education		0.23	0.12	3.57	.061
Expressive vocabulary		0.02	0.04	0.34	.563
Mathematical language		0.32	0.09	11.85	.001
Cardinality (how many)	.33				
Age		-0.001	0.14	<0.001	.994
Female		-0.07	0.15	0.20	.657
Parent education		0.03	0.04	0.59	.444
Expressive vocabulary		0.006	0.01	0.27	.602
Mathematical language		0.16	0.03	25.06	<.001
Cardinality (give-*n*)	.43				
Age		0.43	0.26	2.76	.099
Female		0.49	0.27	3.25	.074
Parent education		0.22	0.07	8.42	.004
Expressive vocabulary		0.02	0.02	0.54	.466
Mathematical language		0.26	0.06	19.62	<.001
Connecting numerals to quantities	.32				
Age		0.67	0.23	8.88	.003
Female		0.15	0.24	0.38	.539
Parent education		-0.02	0.07	0.10	.757
Expressive vocabulary		0.02	0.02	0.78	.379

Mathematical language		0.20	0.05	15.10	<.001
Set comparison	.40				
Age		0.51	0.23	5.15	.025
Female		0.15	0.24	0.38	.539
Parent education		0.03	0.07	0.20	.656
Expressive vocabulary		0.04	0.02	4.90	.029
Mathematical language		0.20	0.05	16.22	<.001
Numeral comparison	.36				
Age		0.74	0.26	7.98	.006
Female		0.48	0.27	3.10	.081
Parent education		0.18	0.08	5.71	.018
Expressive vocabulary		0.01	0.02	0.11	.744
Mathematical language		0.21	0.06	12.97	<.001
Number order	.41				
Age		0.38	0.28	1.76	.188
Female		0.60	0.30	3.95	.049
Parent education		0.14	0.08	2.99	.086
Expressive vocabulary		0.002	0.02	0.01	.946
Mathematical language		0.34	0.06	28.19	<.001
Story problems	.40				
Age		0.22	0.30	0.55	.461
Female		0.17	0.30	0.33	.568
Parent education		0.22	0.09	5.54	.021

Expressive vocabulary		0.08	0.02	9.87	.002
Mathematical language		0.18	0.07	6.99	.009
Subitizing	.20				
Age		0.62	0.22	7.98	.006
Female		-0.31	0.23	1.81	.181
Parent education		0.08	0.06	1.51	.222
Expressive vocabulary		0.03	0.02	2.66	.106
Mathematical language		0.08	0.05	2.70	.103
Formal addition	.11				
Age		0.54	0.27	3.87	.051
Female		-0.002	0.27	<0.001	.996
Parent education		0.06	0.08	0.45	.504
Expressive vocabulary		0.03	0.02	1.55	.216
Mathematical language		0.06	0.06	1.06	.306

Note. $N = 124$.

Mathematical language. In alignment with expectations, mathematical language was significantly related to every complex numeracy skill above and beyond general language ability, including cardinality, connecting numerals to quantities, set comparison, numeral comparison, number order, and story problems (see Table 3). Mathematical language was not significantly related to subitizing or formal addition, also as hypothesized. However, mathematical language was significantly related to each of the three basic skills above and beyond general language ability which were hypothesized to be related to general language ability only, specifically verbal counting, one-to-one correspondence, and numeral identification. It should be noted that when

IQ was included in the model, mathematical language was no longer significantly related to verbal counting ($b = 0.12$, $SE = 0.06$, $F[1, 123.51] = 3.56$, $p = .062$), or story problems, after the Benjamini-Hochberg correction was applied ($b = 0.15$, $SE = 0.07$, $F[1, 123.35] = 4.98$, $p = .027$).

General language ability. Contrary to expectations, with the exception of set comparison and story problems, general language ability was not significantly related to any proposed numeracy skills. Even in each model of basic numeracy skills (verbal counting, one-to-one correspondence, and numeral identification) proposed to be uniquely related to expressive vocabulary only, the coefficient for general language ability was not significant.

In a model predicting set comparison with the additional control of IQ, the coefficient for general language ability was no longer significant ($b = 0.01$, $SE = 0.02$, $F[1, 123.93] = 0.10$, $p = .756$), whereas the coefficient for mathematical language remained significant ($b = 0.18$, $SE = 0.05$, $F[1, 115.66] = 12.56$, $p = .001$). In the model predicting story problems with the additional control of IQ, the coefficient for general language ability was no longer significant ($b = 0.04$, $SE = 0.03$, $F[1, 119.53] = 2.07$, $p = .153$), and the coefficient for mathematical language became nonsignificant after the Benjamini-Hochberg correction was applied, as noted above. All other models with the inclusion of IQ resulted in the same conclusions.

Covariates. Children's age was significantly related to several numeracy skills, including one-to-one correspondence, connecting numerals to quantities, set comparison, numeral comparison, and subitizing. Child gender was a significant factor in the number order model, favoring girls; however, this difference is likely not meaningful as it was the only significant gender difference found and would no longer be significant after adjusting for the number of tests conducted. Finally, parent education was a significant factor in the models for one-to-one correspondence, numeral comparison, story problems, and cardinality as measured by the give-*n* task.

Discussion

Findings from this study generally support the hypothesis that mathematical language would be related to complex numeracy skills above and beyond general language, as mathematical language was significantly related to cardinality, connecting numerals to quantities, set comparison, numeral comparison, number order, and story problems. In contrast, general language ability was not significantly related to basic numeracy skills as hypothesized. In fact, significant associations were present between mathematical language and each of the basic skills that were proposed to be related to general language ability only: verbal counting, one-to-one correspondence, and numeral identification. In each model, mathematical language was significantly related to each specific numeracy skill over and above general language, when also controlling for age, gender, and parent education. General language ability only had significant relations to performance on two numeracy tasks: set comparison and story problems, and those relations did not hold once IQ was added as an additional control. Furthermore, even when IQ was added into the model, mathematical language generally remained significantly related to each proposed numeracy skill. Finally, as expected, both mathematical language and general language ability were unrelated to subitizing and formal addition skills.

The present study is an extension of prior work demonstrating that children's understanding of mathematical language is more proximal to children's numeracy skills, broadly measured, than is general language (Purpura & Reid, 2016; Toll & Van Luit, 2014a, b). With the present results it is evident that mathematical language is more proximal than general language for most specific numeracy skills. A foundation of mathematical language allows children to understand higher-order mathematics concepts and manipulate numbers in complex ways, including comparing quantities and connecting numerals to quantities. The finding that mathematical language is related to the more basic skills, for which general language was proposed to play more of a role (e.g., verbal counting, one-to-one correspondence, numeral identification) was unexpected, and indicates that mathematical language may underlie development of each early numeracy skill. Children with stronger mathematical language skills may be in environments that promote both mathematical language understanding and basic understanding of the names of numbers, which could help children to connect number names with these procedural skills as well as more advanced numeracy skills. Although Purpura and Ganley (2014) found that general vocabulary is related to a broad range of numeracy skills, it is actually mathematical language that is uniquely related to each specific numeracy skill; that is, general language was acting as a proxy for mathematical language not only for broad relations, as noted in previous work (Purpura & Reid, 2016; Toll & Van Luit, 2014b), but for skill-specific relations as well.

Implications

Results from this study have significant implications. Overall, results underscore the importance of mathematical language, as it is more proximal to most numeracy skills than is general language ability. The current study's examination of nuanced relations between mathematical language and specific numeracy skills provides key insight for future research on assessment and intervention development. Specifically, results suggest that measures of mathematical language may be more appropriate to include in assessments of children's mathematical learning than would be measures of general language, as is more typically done. For example, a large body of work has examined the overlap in mathematical learning difficulties and reading difficulties and found that children with comorbid difficulties typically exhibit more significant mathematics difficulties than do children with mathematics difficulties alone (Hanich, Jordan, Kaplan, & Dick, 2001; Jordan & Hanich, 2000). The use of assessments of mathematical language, as opposed to general language measures, may lead to more precise identification of later risk for mathematics difficulties. Given the importance of school-entry mathematics understanding for later achievement (Duncan et al., 2007; Jordan et al., 2009; Watts et al., 2014), improved identification of children at risk for difficulties before these difficulties become manifest is a critical focus for future research.

Beyond implications for assessment, understanding how mathematical language relates to specific numeracy skills above and beyond general language may enable developers of early numeracy interventions to better account for mathematical language within intervention activities. Incorporating scaffolding for the mathematical language demands of various activities may improve the effectiveness of early numeracy interventions. In future intervention development, children's understanding of mathematical language can also be assessed and used as a framework for individualizing instruction on specific numeracy skills.

The importance of mathematical language has been emphasized by educators in recent decades in calls to increase focus on content-specific mathematical vocabulary in preschool and elementary school (Lansdell, 1999; Riccomini, Smith, Hughes, & Fries, 2015), and specific efforts to support clear and concise mathematics language (Hughes, Powell, & Stevens, 2016). Mathematical language, especially quantitative language, is currently scarce in early childhood settings and greater attention is needed to promote this higher-level language content (Rudd, Lambert, Satterwhite, & Zaier, 2008). Results from the current study are in line with these recent efforts that highlight the foundational role of mathematical language in early numeracy skills.

Limitations and Future Directions

This study has a few limitations that must be addressed. First, results are correlational in nature and children's understanding of mathematical language and specific numeracy skills were assessed at only one time point. Thus, care should be taken to not interpret findings causally or assume directionality of the relations. Although it may be possible that children's acquisition of specific numeracy skills enhances their mathematical language understanding, it is more likely that mathematical language underlies numeracy skills, broadly, as has been found in prior experimental work (Purpura, Napoli, Wehrspann, & Gold, 2017). Importantly, the findings from this study provide a foundation for future research to assess longitudinal relations and effects of experimental designs on the relation between mathematical language and specific numeracy skills. Furthermore, additional work is needed to understand how mathematical language and specific numeracy skills develop across preschool and into formal schooling.

Second, the variable for children's understanding of mathematical language in this study encompassed understanding of both quantitative (e.g., "more", "less") and spatial (e.g., "near", "above") words. Thus, it is unclear whether particular types of mathematical language are differentially related to specific numeracy skills (e.g., perhaps spatial language would show the strongest relation to number order because performance on that task requires understanding "before" and "after"). Future research is needed to examine how particular facets of mathematical language (e.g., quantitative, spatial) are individually related to targeted aspects of numeracy skills.

Third, this study included relevant covariates for each model (age, gender, and parental education, as well as IQ in the original models); however, there are other potential covariates that were not included. For instance, aspects of literacy such as print knowledge have been found to be related to numeracy skills above and beyond general language skills (Neumann, Hood, Ford, & Neumann, 2013; Purpura et al., 2011; Purpura & Napoli, 2015), and may have a strong relation with numeral identification in particular. Children's understanding of syntax has also been linked to their mathematics performance (Chow & Ekholm, 2018); however, it is unclear how understanding of syntax is related to preschoolers' understanding of specific numeracy skills, which could be examined in future research. Furthermore, children's classroom and home numeracy environment were not measured in this study. It is possible that individual differences in these environments may underlie some of the similarities in relations observed between domains (basic and complex numeracy skills). The relation between mathematical language and each numeracy skill was examined with specific covariates included, and thus did not include all possible covariates.

Fourth, it should be noted that although the sample of children assessed in this study was representative of the Midwestern community from which it was recruited, it was not representative of children across the United States more broadly. More research at a national scale is needed to make conclusions about relations between these variables across regions of the United States.

Finally, only numeracy skills were assessed in this study. Other aspects of mathematics such as geometry and patterning skills may also be related to mathematical language skills. Future research examining the relation between mathematical language and other aspects of mathematics, as well as spatial tasks, is needed.

Conclusion

This study expands on prior work by examining targeted relations between preschoolers' understanding of mathematical language and specific numeracy skills. In contrast to prior work indicating that general vocabulary is related to a wide range of numeracy skills (Purpura & Ganley, 2014), it is in fact mathematical language that is specifically related to these numeracy skills. Results extend our understanding of the skill-specific relations between mathematical language and numeracy skills in early childhood, in which mathematical language is more proximal to most specific numeracy skills than is general language. These results provide a foundation for future research to develop more precise measures to identify children at risk for mathematics difficulties, as well as incorporate mathematical language within preschool mathematics curricula, in order to prepare children for successful entry into formal schooling.

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